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MASTER THESIS

Option: Computer science

Conflict modeling with the theory of belief functions for database improvement

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Abstract

The theory of belief functions is used for representing uncertain and imprecise information and also for combining several sources' opinions. The conflict appearing in the combination can be computed from a distance measure in the purpose of estimating the relative reliability of each source. This conflict can be managed before the combination step by taking into account the reliabilities of the sources and discounting the related information. This method needs knowledge about the source's degree of reliability, which can be estimated from the related belief functions. In this master thesis, we propose a generalized method for source's reliability estimation taking into account all their belief functions stored in an evidential database and also insuring the same level of reliability for all these belief functions. Sources' reliability degrees are used to discount their belief functions stored in evidential databases as well as the related plausibility functions. These methods are evaluated on real radar data and supplied good results in terms of sources' reliability improvement.

Résumé

La théorie de fonctions de croyance est utilisée pour la représentation des informations incertaines et imprécises, et plus particulièrement pour la combinaison des avis de plusieurs sources (experts). Le conflit apparaissant lors de la combinaison peut être estimé à partir d'une mesure de distance afin d'évaluer la fiabilité relative de la source. Ce conflit peut être géré avant la combinaison en tenant compte des fiabilités des sources et en affaiblissant les informations qu'elles fournissent. Cette méthode nécessite une connaissance préalable des degrés de fiabilité des sources qui peuvent êtres évalués à partir des données qu'elles fournissent. Dans ce rapport de mastère, nous proposons une méthode permettant l'estimation des fiabilités des sources à partir de toutes leurs fonctions de croyance stockées dans des bases de données évidentielles. Nous proposons également d'assurer le même niveau de fiabilité pour toutes ces fonctions de croyance. Les degrés de fiabilité des sources sont utilisés pour affaiblir leurs fonctions de croyance stockées dans des bases de données évidentielles ainsi que leurs fonctions de plausibilité. Ces deux méthodes ont été évaluées sur des données radar et ont permis d'améliorer les fiabilités de leurs sources.

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Introduction

Motivation and purpose

Decision making is more and more difficult when using imperfect data where available information are imprecise, uncertain and even not available. Usually decision is made on the bases of precise and certain data but available information are not always so. Many theories manage uncertainty such as probability theory, fuzzy set theory (Zadeh, 1965), possibility theory (Dubois and Prade, 1988) and the theory of belief functions (Dempster, 1967) (Shafer, 1976). This latter theory is a strong tool for representing information of several levels of uncertainty from certain to uncertain and missing data.

Within imperfect environment, combining several imperfect information helps users and decision makers to reduce the degree of uncertainty by confronting several opinions. The theory of belief functions presents a strong framework for combination which is not provided by probability theory and fuzzy sets theory. Thus, the theory of belief functions is the more appropriate theory for handling uncertainty and also combining several experts' opinions.

In the other side, databases are used to store a high quantity of structured data which are usually perfect. Most of the time, available data are imperfect, thus the use of evidential databases becomes a necessity in order to store both certain and uncertain data.

In many fields such as target recognition the number of evidential databases is great, and they store most of the time the same information provided by different sources. Therefore, integrating evidential databases reduces the quantity of data to be stored and also helps decision makers when handling all available information because they

will use only an integrated evidential database rather than many separated ones.

When combining uncertain information stored in different evidential databases, a conflict may arise due to the conflict between their sources. This conflict may be managed when combining by the use of combination rules which redistribute it with different manners on the focal elements like (Dubois and Prade, 1988), (Murphy, 2000), (Smets and Kennes, 1994), (Martin and Osswald, 2007a) and (Yager, 1987). The main reason of the conflict apparition is the relative reliability of the sources, thus it can be avoided before combining by taking into account sources' reliabilities. Discounting operator is used to balance evidential information with their source's degree of reliability, thus no conflict apparition after combining and even if it appears it will be bearable.

To discount evidential information, source's reliability is supposed to be known although it is not always the case. The main purpose of this master report is to propose a generalized method which aims to estimate source's reliability degree on the bases of all its information stored in an evidential database. Source's reliability has to be estimated in order to discount its evidential information before combining them with other evidential information supplied by other sources. The purpose of combining several evidential information is to merge evidential databases.

Master contribution

In this dissertation, we propose a twofold contribution:

- Theoretical contribution: We define the theoretical aspects of source's reliability estimation on the bases of all its evidential information which are stored in an evidential database in the purpose of integrating the latter with other ones.
- Practical contribution: Once source's reliability is estimated, it will be used to discount all its evidential information. This method is applied on real databases containing radar data.

Master organization

The outline of the master is as follows:

In Chapter 1, we recall basic concepts of the theory of belief functions as well as tools given by this theory which aims to combine several evidential information. Methods used to solve the conflict which may appear in the combination are also presented.

In Chapter 2, we will present some distances used in the theory of belief functions, then we will explain some methods of reliability estimation and learning. Once source's reliability is estimated or learned, evidential information can be discounted therefore some methods of discounting are also summarized in this chapter.

Chapter 3 is devoted to describe our method of source's reliability estimation from all its evidential information.

Finally, Chapter 4 gives our method's results of test on real radar data.

1

Theory of belief functions

1.1 Introduction

The theory of belief functions was first introduced by Dempster in 1967 (Dempster, 1967) and Shafer in 1976 (Shafer, 1976) for quantifying beliefs and that is why it is also called Dempster-Shafer theory. Later P. Smets proposed an other interpretation of this theory: The Transferable Belief Model (TBM).

It is a mathematical theory that extends probability theory by giving up measures' additivity constraint, therefore belief masses are assigned instead of probability values. Belief masses can also be affected to subsets of elements contrary to probabilities which are affected to atomic elements only.

This theory allows the representation of uncertainty and also the combination of different pieces of evidence provided by different sources in order to obtain a belief function that takes into account all available evidences.

In the TBM, belief is divided into two levels: a credal level [1] where beliefs are quantified, represented (the static part) and combined (the dynamic part), and a pignistic level [2] where beliefs are used for decision making with regard to the risk and the earnings associated to this decision.

In this chapter, we will first introduce some basic concepts of this theory like frame of discernment, belief functions and transformations of belief functions in order to see how belief is represented.
Second, we will introduce some tools given by this theory like conditioning, coarsening

[1] from Latin "credo" means "I believe"
[2] from Latin "pignus" means "a bet"

and refinement, combination and conflict solving. Finally, we will present the pignistic level and decision making.

1.2 Frame of discernment

Let $\Omega = \{\omega_1, \ldots, \omega_k, \ldots, \omega_n\}$ be a set of n non empty and mutually exclusive elementary hypotheses related to a given problem. These hypotheses correspond to the possible values of an object's attribute that we attend to reach the real value. In other words, we look for the actual value ω_0 of an attribute in Ω.
Ω is called **frame of discernment** or **universe of discourse**.

Our purpose is to determine which element of the frame of discernment Ω corresponds to the actual value ω_0.

Under a **closed world assumption**, we suppose that Ω is exhaustive therefore it includes all the possible values of the target attribute, thus the real value ω_0 is inevitably one element of Ω.
This assumption was used in Shafer's model (Dempster, 1967) but it is always difficult to enumerate from the beginning all the hypotheses related to a given problem, that is why Smets (Smets, 1990) introduced the **open world assumption**.

Under an open world assumption, Ω is not necessarily exhaustive therefore we admit that the actual value ω_0 can be an unknown hypothesis which is not mentioned into the frame of discernment Ω.

1.3 Belief functions

Let 2^Ω be a set of all subsets made up of hypotheses and unions of hypotheses from Ω. 2^Ω is called **power set** and defined as follows:

$$2^\Omega = \{A : A \subseteq \Omega\} \tag{1.1}$$

Every element of 2^Ω is a **proposition** or an **event**.

Example 1.1 Let us consider a problem of air target classification. A target can be a plane, an helicopter or a missile, therefore our frame of discernment Ω is constituted of helicopter H, missile M and plane P:
$\Omega = \{H,\ M,\ P\}$.
The corresponding power set is:
$2^\Omega = \{\emptyset,\ H,\ M,\ P,\ H \cup M,\ H \cup P,\ M \cup P,\ H \cup M \cup P\}$. ■

1.3.1 Basic belief assignment

Belief functions are represented in different ways but the most common one is the basic belief assignment (bba) which represents our evidence held by an agent (or a 'belief holder'). This agent can be a robot, a sensor, an expert...

A bba is a mapping m^{Ω} from $2^{\Omega} \rightarrow [0,1]$ such that:

$$B \mapsto m^{\Omega}(B) \tag{1.2}$$

$m^{\Omega}(B)$ is called a basic belief mass (bbm) and it is the expert's degree of faith on B. It represents the degree of confidence assigned by an expert to B.

A basic belief assignment supplied by a source s has to verify:

$$\sum_{B \subseteq \Omega} m_s^{\Omega}(B) = 1 \tag{1.3}$$

Example 1.2 Let us take the same problem of air target classification, the sensor assigned these masses:
$m_s^{\Omega}(P) = 0.6$
$m_s^{\Omega}(M) = 0.4$
It means that the source (sensor) believes that the detected object is a plane with a degree of belief 0.6 and that it is a missile with a degree of 0.4. ■

A bbm assigned to a proposition B from 2^{Ω}, for example $B = \{M \cup P\}$, $m^{\Omega}(B)$ represents explicitly the doubt between the component hypotheses of B, it means that we have a belief on B which is $m^{\Omega}(B)$ but we cannot deduct $m^{\Omega}(M)$ and $m^{\Omega}(P)$ from it.

A subset B of Ω having a strictly positive mass, $m^{\Omega}(B) > 0$, is called **focal element**. For example $m^{\Omega}(H \cup P) = 0.2$ implies that $\{H \cup P\}$ is a focal element.

The couple (F, m^{Ω}) is called **body of evidence** such that F is the set of all focal elements of the bba m^{Ω}.

The core φ of a bba is the union of all its focal elements and is defined as follows:

$$\varphi = \bigcup_{A : m^{\Omega}(A) > 0} A \tag{1.4}$$

The mass $m^{\Omega}(\Omega)$ is the part of belief assigned to the hole frame of discernment. It represents the amount of our **total ignorance**.

The mass $m^{\Omega}(\emptyset)$ represents the degree of belief that the real value of the target attribute is not enumerated in the frame of discernment Ω. In Shafer's model (Shafer, 1976), a condition of normalized bba is expressed by:

$$m^{\Omega}(\emptyset) = 0$$

In this case a closed world assumption is admitted and such bba is called a **normalized basic belief assignment**.

In the TBM this condition is not required because of the open world assumption, therefore a non strictly positive mass can be allocated to the empty set $m^{\Omega}(\emptyset) \geq 0$. This value is allocated to the hypothesis that the actual value ω_0 might not lie in Ω.

A non normalized bba may be transformed into a normalized one using an operator of normalization defined as follows:

$$\begin{cases} m'^{\Omega}(A) = \dfrac{m^{\Omega}(A)}{1 - m^{\Omega}(\emptyset)} & \forall A \subseteq \Omega \\ m'^{\Omega}(\emptyset) = 0 \end{cases} \tag{1.5}$$

Example 1.3 For the same frame of discernment $\Omega = \{H,\ M,\ P\}$. Suppose that the sensor expressed this piece of evidence:
$m^{\Omega}(P) = 0.6$
$m^{\Omega}(P \cup M) = 0.2$
$m^{\Omega}(\emptyset) = 0.2$
The normalization of this bba m^{Ω} gives:
$m'^{\Omega}(P) = 0.75$
$m'^{\Omega}(P \cup M) = 0.25$
$m'^{\Omega}(\emptyset) = 0$
$\{P\}$ and $\{P \cup M\}$ are focal elements, therefore $F = \{\{P\},\ \{P \cup M\}\}$ is the set of focal elements and (F, m^{Ω}) is the body of evidence.
The core of the bba m^{Ω} is: $\varphi = \{P\} \cup \{P \cup M\} = \{P \cup M\}$. ■

1.3.2 Special belief functions

In this section, we shall present some particular bbas.

Categorical belief function

A categorical belief function is a normalized belief function which has an unique focal element different from Ω and $\emptyset$. This belief function can be defined as follows:

$$m^{\Omega}(A) = \begin{cases} 1 & \text{if } A = A^{\star} \subset \Omega \\ 0 & \forall A \subseteq \Omega \text{ and } A \neq A^{\star} \end{cases} \tag{1.6}$$

Example 1.4 Suppose that we received a piece of evidence showing that the air target was either a plane or a missile, the corresponding bba is a categorical belief function:
$m^{\Omega}(\{P \cup M\}) = 1$.

■

When all sources supply categorical bbas, the TBM corresponds to the classical propositional logic.

We distinguish two particular cases of categorical bbas: vacuous belief function and contradictory belief function.

Vacuous belief function (VBF)

A vacuous belief function is a particular categorical belief function focused on Ω. It means that a VBF is a normalized bba having an unique focal element which is Ω. Its corresponding bba is defined as follows:

$$m^{\Omega}(A) = \begin{cases} 1 & \text{if } A = \Omega \\ 0 & \text{otherwise} \end{cases} \tag{1.7}$$

This belief function allows expressing the total ignorance of a source.

Example 1.5 Suppose that the sensor was unable to identify the air target, therefore the situation of total ignorance arises. The bba supplied by this sensor will be defined as follows:
$m^{\Omega}(\Omega) = 1$, $m^{\Omega}(A) = 0 \quad \forall A \subset 2^{\Omega}$ and A$\neq \Omega$.
In this case the unit is attributed to the whole universe. There is no hypothesis more possible than others. ■

Contradictory belief function

A contradictory belief function is a particular categorical belief function focused on $\emptyset$. It means that a contradictory belief function is a non normalized bba having an unique focal element which is the empty set "$\emptyset$". Its corresponding bba is defined as follows:

$$m^{\Omega}(A) = \begin{cases} 1 & \text{if } A = \emptyset \\ 0 & \text{otherwise} \end{cases} \tag{1.8}$$

Such bba means that the value ω_0 of the target variable is an hypothesis which is not mentioned in Ω, therefore Ω is not exhaustive. The open world assumption is admitted in this case.

Example 1.6 Suppose that the sensor detected some thing never seen before. The detected air target is not a plane, a missile nor an helicopter. The bba supplied by this sensor will be defined as follows:
$m^{\Omega}(\emptyset) = 1,\ m^{\Omega}(A) = 0 \quad \forall A \subset 2^{\Omega}$ and $A \neq \emptyset$.
In this case the unit is attributed to the empty set. No hypothesis from the frame of discernment is ω_0. ■

Dogmatic basic belief assignment

A dogmatic bba focused on Ω is a bba such that Ω is not a focal element; its corresponding bba is defined as follows:

$$m^{\Omega}(\Omega) = 0 \tag{1.9}$$

Example 1.2 is an example of a dogmatic basic belief assignment.

Bayesian belief function

A Bayesian belief function is a bba such that all focal elements are singletons; its corresponding bba is defined as follows:

$$\begin{cases} m^{\Omega}(A) \in [0,1] & \text{if } \mid A \mid = 1 \\ m^{\Omega}(A) = 0 & \text{otherwise} \end{cases} \tag{1.10}$$

As all focal elements are singletons, this belief function is a **probability distribution**.

Example 1.7 Let us take the same example of air target classification and suppose that the sensor assigned these masses:
$m^{\Omega}(\{P\}) = 0.2$
$m^{\Omega}(\{M\}) = 0.5$
$m^{\Omega}(\{H\}) = 0.3$
This bba is a Bayesian belief function which corresponding probability distribution is:
$p(P) = 0.2$, $p(M) = 0.5$ and $p(H) = 0.3$. ■

Consonant belief function

A consonant belief function is a bba such that all focal elements are nested ($A_1 \subset A_2 \subset \ldots \subset \Omega$).

Example 1.8 This following bba is a consonant belief function:
$m^{\Omega}(M) = 0.2$
$m^{\Omega}(M \cup H) = 0.4$
$m^{\Omega}(M \cup H \cup P) = 0.4$
This bba has common characteristics with possibilities. Plausibility and possibility functions are equivalent in this case. ■

Certain belief function

A certain belief function is a categorical belief function (a bba with an unique focal element) such that its focal element is a singleton. This bba corresponds to the case of total certainty as the unit is attributed to only one hypothesis. Its corresponding bba is defined as follows:

$$m^{\Omega}(A) = \begin{cases} 1 & \text{if } A = A^{\star} \in \Omega \\ 0 & \forall A \subseteq \Omega \text{ and } A \neq A^{\star} \end{cases} \tag{1.11}$$

Example 1.9 The sensor confirms that detected air target is a plane therefore:
$m^{\Omega}(\{P\}) = 1.$ ■

Simple support function

A simple support function (SSF) is a belief function which has an unique focal element which is not the frame of discernment Ω. This unique focal element is called **the focus of the simple support function**. This function is defined as follows (Shafer, 1976) and (Smets, 1995):

$$m^{\Omega}(X) = \begin{cases} \omega & \text{if } X = \Omega \\ 1 - \omega & \text{if } X = A \text{ for some } A \subset \Omega \\ 0 & \text{otherwise} \end{cases} \tag{1.12}$$

where A is the **focus** of the SSF and $\omega \in [0, 1]$.

This SSF is denoted A^{ω} where ω is the basic belief mass (bbm) affected to the frame of discernment Ω and the complement of ω to 1 is the bbm affected to A.

Example 1.10 Suppose having the frame of discernment $\Omega = \{H,\ M,\ P\}$.
Assume that this bba is defined on Ω:
$m^{\Omega}(P \cup H) = 0.6$
$m^{\Omega}(\Omega) = 0.4$
m^{Ω} is a SSF with $\{P \cup H\}$ its focus. ■

1.4 Transformations of belief functions

We can deduct other functions from basic belief masses. These functions represent differently the same piece of information and they are used especially for computing. They are also a mapping from 2^Ω to $[0,1]$ but the sum of all focal elements' masses is not necessarily equal to 1.

1.4.1 Belief (or credibility) function

A belief function, noted **bel**, is the minimal degree of belief justified by available information.

Unlike bbas which assign an amount of belief to each subset A of Ω, the credibility of a subset, $bel(A)$, is the total amount of belief assigned to subsets implying A. bel(A) is obtained by summing masses of subsets of A. The belief function is defined by:

$$\begin{aligned} bel : 2^\Omega &\to [0,1] \\ A &\mapsto \sum_{B \subseteq A, B \neq \emptyset} m^\Omega(B) \end{aligned} \tag{1.13}$$

$$m^\Omega(B) = \sum_{B \subseteq A} (-1)^{|A|-|B|} bel(B) \tag{1.14}$$

$$m^\Omega(\emptyset) = 1 - bel(\Omega) \tag{1.15}$$

As the empty set is included in both of A and $\bar{A}$, it is regarded from the sum.

Properties:

- **Sub-additivity:** $bel(A) + bel(\bar{A}) \leq 1$
- **Monotonicity:** $A \subseteq B \Rightarrow bel(A) \leq bel(B)$
- $bel(A) + bel(B) \leq bel(A \cup B)$
- $bel(\Omega) = 1$ and $bel(\emptyset) = 0$ under the closed world assumption

1.4.2 Plausibility function

The plausibility function is noted **pl**. The plausibility of an event $B \subseteq \Omega$ is the maximum amount of belief that could be given to B. It is measured by summing masses of propositions compatible with B. Plausibility function is defined as:

$$\begin{aligned} pl : 2^\Omega &\to [0,1] \\ A &\mapsto \sum_{A \cap B \neq \emptyset} m^\Omega(B) \end{aligned} \tag{1.16}$$

Also

$$pl(A) = 1 - bel(\bar{A}) \tag{1.17}$$

$m^{\Omega}(A)$ may be derived from $pl(A)$ as follows:

$$\begin{cases} m^{\Omega}(A) = \sum_{A \subseteq B} (-1)^{|B|-|A|+1} pl(\bar{A}) \\ m^{\Omega}(\emptyset) = 1 - pl(\Omega) \end{cases} \tag{1.18}$$

Properties:

- **Over-additivity:** $pl(A) + pl(\bar{A}) \geq 1$
- **Monotonicity:** $A \leq B \Rightarrow pl(A) \leq pl(B)$
- $pl(A \cup B) \leq pl(A) + pl(B)$
- $bel(A) \leq pl(A)$

1.4.3 Commonality and implicability functions

Commonatily and implicability functions are also transformations of belief functions. These functions have no signification but they are useful to simplify computation. The **implicability function** is defined as follows:

$$\begin{aligned} & b : 2^{\Omega} \to [0,1] \\ & A \mapsto \sum_{B \subseteq A} m^{\Omega}(B) = bel(A) + m^{\Omega}(\emptyset) \end{aligned} \tag{1.19}$$

The **commonality function** is defined as follows:

$$\begin{aligned} & q : 2^{\Omega} \to [0,1] \\ & A \mapsto \sum_{A \subseteq B} m^{\Omega}(B) \end{aligned} \tag{1.20}$$

Example 1.11 In table 1.1, we present the bba given by a sensor (example 1.1) with the corresponding belief, plausibility, implicability and commonality functions.

■

Commonality function is used for combining [3] a great number of belief functions (n bbas with $n > 2$) by computing the commonality function from each bba using equation (1.20) then, multiplying all these commonality functions as follows:

$$q(A) = \prod_{j=1}^{n} q_j(A) \qquad \forall A \subseteq \Omega \tag{1.21}$$

[3] Combination will be discussed later in Section 1.7.1

Table 1.1: Belief, plausibility, implicability and commonality functions

	$\mathbf{m^\Omega}$	**bel**	**pl**	**b**	**q**
$\emptyset$	0.1	0	0	1	1
P	0.05	0.0556	0.44	0.15	0.44
H	0.3	0.333	0.78	0.4	0.78
$P \cup H$	0.2	0.6111	0.84	0.65	0.38
M	0.06	0.0667	0.35	0.16	0.35
$M \cup P$	0.01	0.1333	0.6	0.22	0.19
$H \cup M$	0.1	0.5111	0.85	0.56	0.28
Ω	0.18	1	0.9	1	0.18

Finally, combined bba is obtained by converting back the multiplied commonality function to bba using the following equation:

$$m^\Omega(A) = \sum_{A \subseteq B} (-1)^{|B-A|} q(B) \tag{1.22}$$

Example 1.12 In table 1.2, we present three bbas given by three different sensors but referring to the same object (example 1.2), these three bbas are combined using commonality functions.

Table 1.2: Combining using commonality functions

	$\mathbf{m_1^\Omega}$	$\mathbf{m_2^\Omega}$	$\mathbf{m_3^\Omega}$	$\mathbf{q_1}$	$\mathbf{q_2}$	$\mathbf{q_3}$	$\mathbf{q}$	$\mathbf{m^\Omega}$
$\emptyset$	0.1	0	0	1	1	1	1	0.833
P	0.05	0.3	0.5	0.44	0.6	0.5	0.132	0.132
H	0.3	0.4	0	0.78	0.7	0	0	0
$P \cup H$	0.2	0.1	0	0.38	0.3	0	0	0
M	0.06	0	0.5	0.35	0.2	0.5	0.035	0.035
$M \cup P$	0.01	0	0	0.19	0.2	0	0	0
$H \cup M$	0.1	0	0	0.28	0.2	0	0	0
Ω	0.18	0.2	0	0.18	0.2	0	0	0

■

1.5 Conditioning

When handling a bba given by a source s, a new evidence arises confirming that a proposition A is true (meaning that the actual value ω_0 is in the subset A). Therefore, the mass allocated to A has to be reallocated in order to take consideration of this new evidence. This is realized by the conditioning operator. Conditioning a bba m^{Ω} over a subset A$\subseteq \Omega$ consists on restricting the frame of possible propositions 2^{Ω} to the set of subsets having a non empty intersection with A. Therefore the mass affected to $C \subseteq \Omega$ is allocated to $\{C \cap A\}$. The obtained bba, result of the conditioning, is noted $m^{\Omega}_{[A]} : 2^{\Omega} \rightarrow [0,1]$ such that (Smets and Kruse, 1997):

$$m^{\Omega}_{[A]}(C) = \begin{cases} 0 & \text{for } C \not\subseteq A \\ \sum\limits_{X \subseteq \bar{A}} m^{\Omega}(C \cup X) & \text{for } C \subseteq A \end{cases} \tag{1.23}$$

This conditioning rule is not normalized because a non null mass can be attributed to the empty set.

Under a closed world assumption, the mass attributed to the empty set must be null. In this case Dempster's rule of conditioning seems to be a good solution (Smets and Kennes, 1994) and it is defined as follows:

$$m^{\Omega}_{[A]}(c) = \begin{cases} \dfrac{\sum\limits_{X \subseteq \bar{A}} m^{\Omega}(C \cup X)}{1 - \sum\limits_{X \subseteq \bar{A}} m^{\Omega}(X)} & \text{for } C \subseteq A \\ 0 & \text{for } C \not\subseteq A \text{ or } C = \emptyset \end{cases} \tag{1.24}$$

Example 1.13 For the same example (air target classification), the sensor provides the following bba:

$m^{\Omega}(\{H \cup P\}) = 0.7.$

$m^{\Omega}(\Omega) = 0.3$

Suppose now that an expert has a certain evidence that the air target is not an helicopter. According to this expert, the truth is $B = \{P \cup M\}$. If we apply the conditioning rule over B we obtain:

$m^{\Omega}_{[B]}(\{P\}) = 0.7$

$m^{\Omega}_{[B]}(\{P \cup M\}) = 0.3$

Therefore the mass $m^{\Omega}(\{H \cup P\}) = 0.7$ is transferred to:

$m^{\Omega}(\{H \cup P\} \cap \{P \cup M\}) \Rightarrow m^{\Omega}_{[B]}(\{P\}) = 0.7$ ■

1.6 Coarsening and refinement

Belief functions are provided by different sources which may express their beliefs on different but compatible frames of discernment. Coarsening and refinement are dedicated to establish relationships between different and compatible frames of discernment in order to express beliefs on any one of them.

Let Θ and Ω be two frames of discernment. Ω is a refinement of Θ if it is obtained by splitting all or some hypotheses from Θ (Shafer, 1976). Conversely, Θ is a coarsening of Ω obtained by grouping together hypotheses of Θ. Let us define a refining σ which is a mapping from $2^{\Theta} \rightarrow 2^{\Omega}$ satisfying:

$$\begin{cases} \sigma(\theta) \neq \emptyset & \forall \theta \in \Theta \\ \sigma(\theta) \cap \sigma(\theta') = \emptyset & \text{if } \theta \neq \theta' \\ \bigcup_{\theta \in \Theta} \sigma(\theta) = \Omega \end{cases} \tag{1.25}$$

For each $A \in \Theta$, $\sigma(A)$ is obtained by splitting the elements of A in Ω (Shafer, 1976).

Ω is a **refinement** of Θ and Θ is a **coarsening** of Ω.

Example 1.14 Let us continue with the same example 1.1, $\Theta = \{helicopter,\ missile,\ plane\}$.
A possible refinement of Θ is:
$\Omega = \{big\ helicopter,\ small\ helicopter,\ Missile,\ big\ plane,\ small\ plane\}$.
where:
$\sigma(helicopter) = \{big\ helicopter,\ small\ helicopter\}$
$\sigma(missile) = \{missile\}$
$\sigma(plane) = \{big\ plane,\ small\ plane\}$
Also, Θ is a **coarsening** of Ω. ■

1.7 Combination

In this section, we will consider the case where two sources express their beliefs about the same problem under the same frame of discernment. These two belief functions have to be combined in order to summarize them in only one.

1.7.1 Combination rules

Let S_1 and S_2 be two distinct and independent sources providing two different bbas m_1^{Ω} and m_2^{Ω} defined on the same frame of discernment Ω. Combining these belief functions induces a third one m_3^{Ω} defined on the same frame of discernment Ω. To combine

these bbas several rules can be used. Rules presented in this section are based on the assumption that sources are independent and distinct (Smets, 1993).

There are a great number of combination rules (Smets, 2007), whereas we present here some of them. The first combination rule was proposed by Dempster in (Dempster, 1967) and is defined for two distinct bbas m_1^Ω and m_2^Ω as follows:

$$m_{1\oplus 2}^\Omega(A) = (m_1^\Omega \oplus m_2^\Omega)(A) = \begin{cases} \dfrac{\sum\limits_{B\cap C=A} m_1^\Omega(B)\times m_2^\Omega(C)}{1-\sum\limits_{B\cap C=\emptyset} m_1^\Omega(B)\times m_2^\Omega(C)} & \forall A\subseteq\Omega,\ A\neq\emptyset \\ 0 & \text{if } A=\emptyset \end{cases} \tag{1.26}$$

A null mass is affected to the empty set, therefore this rule is normalized and works under the closed world assumption where all the possible hypotheses of the studied problem are supposed to be enumerated on Ω.
We notice that $m(\emptyset)$ must be different from 1, thus combining contradictory certain bbas must be avoided.

In order to solve the problem enlightened by Zadeh's counter example in (Zadeh, 1978) where Dempster's rule of combination produced unsatisfactory results, many combination rules appeared.

Smets (Smets, 1990) proposed to use an open world assumption where a non null mass can be affected to the empty set representing the degree of belief that the attribute's real value is not enumerated in Ω. Therefore the conjunctive rule of combination (CRC) for two bbas m_1^Ω and m_2^Ω is defined as follows:

$$m_{1\textcircled{\cap}2}^\Omega(A) = (m_1^\Omega \textcircled{\cap} m_2^\Omega)(A) = \sum_{B\cap C=A} m_1^\Omega(B)\times m_2^\Omega(C) \tag{1.27}$$

In (Dempster, 1967), the mass affected to the empty set ($m(\emptyset)$) is interpreted as the amount of conflict between sources of m_1^Ω and m_2^Ω.

The CRC is used only when both sources are known to be reliable. Smets (Smets, 1990) proposed also to use the disjunctive rule of combination when an unknown source is unreliable. The disjunctive rule of combination (DRC) for two bbas m_1^Ω and m_2^Ω is defined as follows:

$$m_{1\textcircled{\cup}2}^\Omega(D) = (m_1^\Omega \textcircled{\cup} m_2^\Omega)(D) = \sum_{B\cup C=D} m_1^\Omega(B)\times m_2^\Omega(C) \tag{1.28}$$

Yager in (Yager, 1987) interpreted $m(\emptyset)$ as the amount of total ignorance, therefore it is affected to Ω. Yager's rule of combination is defined for two bbas m_1^Ω and m_2^Ω as

follows:

$$\begin{cases} m_Y^\Omega(X) = m_{1\textcircled{\cap}2}^\Omega(X) & \forall X \in 2^\Omega,\ X \neq \Omega \text{ and } X \neq \emptyset \\ m_Y^\Omega(\Omega) = m_{1\textcircled{\cap}2}^\Omega(\Omega) + m_{1\textcircled{\cap}2}^\Omega(\emptyset) \\ m_Y^\Omega(\emptyset) = 0 \end{cases} \tag{1.29}$$

Dubois and Prade's solution (Dubois and Prade, 1988) was to affect the mass resulting from the combination of conflicting focal elements to the union of these subsets.

$$\begin{cases} m_{DP}^\Omega(X) = m_{1\textcircled{\cap}2}^\Omega(X) + \sum\limits_{A\cap X=\emptyset,\ A\cup X=B} m_1^\Omega(X) m_2^\Omega(A) \quad \forall A \subseteq \Omega,\ A \neq \emptyset \\ m_{DP}^\Omega(\emptyset) = 0 \end{cases} \tag{1.30}$$

Finally, Murphy in (Murphy, 2000) proposed the average of bbas as a combination rule. Therefore, for each focal element X of M bbas, the combined one is defined as follows:

$$m_{Mean}^\Omega(X) = \frac{1}{M} \sum_{i=1}^{M} m_i^\Omega(X) \tag{1.31}$$

Example 1.15 In table 1.3, we present two bbas given by two different sensors, these bbas are combined using different combination rules.

Table 1.3: Combining using different combination rules

	$\mathbf{m_1^\Omega}$	$\mathbf{m_2^\Omega}$	$\mathbf{m_{1\oplus 2}^\Omega}$	$\mathbf{m_{1\textcircled{\cap}2}^\Omega}$	$\mathbf{m_Y^\Omega}$	$\mathbf{m_{DP}^\Omega}$	$\mathbf{m_{Mean}^\Omega}$
$\emptyset$	0.1	0	0	0.292	0	0	0.05
P	0.05	0.3	0.209	0.148	0.148	0.178	0.175
H	0.3	0.4	0.5819	0.412	0.412	0.452	0.35
$P \cup H$	0.2	0.1	0.1102	0.078	0.078	0.198	0.15
M	0.06	0	0.0169	0.012	0.012	0.012	0.03
$M \cup P$	0.01	0	0.0028	0.002	0.002	0.002	0.005
$H \cup M$	0.1	0	0.0282	0.02	0.02	0.044	0.05
Ω	0.18	0.2	0.0508	0.036	0.328	0.096	0.19

■

1.7.2 Static and dynamic combination

Smets in (Smets, 2007) distinguish static fusion from dynamic fusion. In static fusion, beliefs are combined simultaneously but in dynamic fusion, beliefs are collected sequentially one by one like detecting an approaching object where a belief is collected from different distances of this object.

Example 1.16 For the same problem of air target classification, two sensors provide two bbas defined on $\Omega = \{H,\ M,\ P\}$. Combining these bbas is a static fusion because they are collected at the same time by both sensors; therefore sensors provided simultaneously these pieces of evidence.
If the same sensor s_1 detected the same target at different times and gave m_1^Ω at t_1 and m_2^Ω at t_2, combining m_1^Ω and m_2^Ω is a dynamic fusion. ■

1.8 Conflict solving

When combining two beliefs, a non null mass may be affected to the empty set ($m(\emptyset) > 0$), $m(\emptyset)$ is interpreted as the amount of conflict (Dempster, 1967). Martin et al. in (Martin et al., 2008), do not consider $m(\emptyset)$ as an indicator about the conflict between sources because it contains an amount of auto-conflict owed on the characteristic of the combination rule which is not idempotent.

The main reason of the conflict apparition when combining two bbas is the relative reliability of their sources. Some degree of conflict can be tolerated but in some cases this degree can be alarming and must be eliminated. To eliminate the conflict, its issue must be detected.

1.8.1 Resolving conflict in the TBM

The TBM conjunctive rule of combination is the only combination rule that keeps $m(\emptyset) > 0$, therefore it affects the conflict to the empty set and uses an expert system to solve the problem when possible (Smets, 2007). A non empty mass affected to the empty set alarms the expert system who tries to look at the origins of the conflict and finds the appropriate solution without affecting belief functions them selves. It may use different combination rules according to conflict's issue.

1.8.2 Conflict due to sources' unreliability

When combining two belief functions provided by different sources, one of these sources may be totally or partially unreliable.

If the knowledge of which source is unreliable cannot be possible, the disjunctive rule of combination performs well.
Discounting operator can be used when at least one source is partially unreliable and its degree of reliability is known or can be computed, this operator will be detailed in a next chapter.

Table 1.4 summarizes methods used to solve the conflict appearing when at least one of the sources is unreliable.

Table 1.4: Methods used to solve the conflict due to source's unreliability

Sources reliability	**Solutions**
An unknown source is unreliable	DRC
A known source is unreliable	Discounting the corresponding bba
Both sources are reliable	CRC and derived rules

1.8.3 Sources reporting about different objects

In that case, sources must be grouped according to objects they report about. Combining is performed on bbas which reports about the same object at the same time.

Example 1.17 Suppose having bbas given in table 1.5, and defined on $\Omega = \{M,\ H,\ P\}$. These bbas describe targets detected by different sensors namely s_1, s_2 and s_3 at different periods. All these bbas cannot be combined together because their sources are

Table 1.5: Bbas given by different sources at different periods

Source	**View**	**Bba**
s_1	v_1	$M(1)$
s_3	v_1	$M \cup P(0.5),\ M(0.5)$
s_1	v_2	$\Omega(0.5)$
s_2	v_1	$M \cup H(0.5),\ \Omega(0.5)$
s_3	v_2	$H(1)$
s_2	v_2	$H(O.5),\ P(0.5)$

reporting about different objects. Therefore, only bbas detected on the same view can be combined.
In one side, bbas of the first, second and fourth lines of table 1.5 can be combined and in the other side, bbas of the third, fifth and sixth lines can also be combined. ■

1.8.4 Conflict due to closed world assumption

Suppose m_1^Ω and m_2^Ω two bbas defined on 2^Ω to be combined under the closed world assumption, a conflict can appear when Ω is not exhaustive therefore some alternatives

have been forgotten when establishing Ω or some alternatives changed in time.

The solution is to add an extra alternative $\omega^\star$ to Ω that includes all those hypotheses not mentioned in Ω. Therefore $\Omega^\star = \Omega \cup \{\omega^\star\}$ is the extended frame and for every $A \subseteq \Omega$ the mass attributed to A, $m^\Omega(A)$, must be allocated to $\{A \cup \omega^\star\}$ ($m^\Omega(\{A \cup \omega^\star\}) = m^\Omega(A)$).
In this case, conflict disappears completely as $\{\omega^\star\}$ belongs to all prepositions of both bbas therefore no empty intersection can be detected.

A problem can appear when making decision, therefore $\omega^\star$ may be the most probable solution. In this case we should condition the bba on Ω before performing pignistic transformation.

1.8.5 Use of combination rules

One solution to hide the conflict appearing in the combination is to use a combination rule which redistributes it regardless of its causes.

Most of rules presented in Section 1.7.1 are based on the conjunctive rule of combination but they are different in the manner of conflict redistributing.
Murphy's combination rule is the only rule (from presented ones) which is not based on the conjunctive rule of combination and conflict does not appear if bbas involved in the combination are normalized. Table 1.6 summarizes methods of conflict redistributing.

Table 1.6: Conflict redistribution methods

Combination rule	**Characteristic of Ω**	**Conflict redistribution**
Dempster's rule of combination	Exhaustive	The conflict is redistributed proportionally on the subsets
Conjunctive rule of combination	Not exhaustive	Conflict is not redistributed
Yager's rule of combination	Exhaustive	$m(\emptyset)$ is affected to Ω
Dubois et Prade's rule of combination	Exhaustive	Masses resulting of conflicting focal elements combination are affected to the union of these focal elements

1.9 Decision making using pignistic probabilities

The purpose of using belief functions is decision making. Decision making consists on choosing one hypothesis ω_0 from Ω.

In the TBM, this process is done using pignistic probabilities distribution (Smets, 2005) noted $BetP\{m^\Omega\}$ calculated from m^Ω for each hypothesis ω_i from Ω.
The pignistic transformation consists on dividing the mass attributed to a proposition A on the hypotheses which train it.

The pignistic probability is defined as follows:

$$\begin{aligned} BetP\{m^\Omega\}: \quad & \Omega \to [0,1] \\ & \omega \mapsto \frac{1}{(1-m^\Omega(\emptyset))} \times \sum_{B \subseteq \Omega,\ \omega_k \in B} \frac{m^\Omega(B)}{\mid B \mid} \end{aligned} \tag{1.32}$$

When making decision, the hypothesis having the highest $BetP\{m^\Omega\}$ is chosen.

Example 1.18 Let us finish with the same problem described in example 1.1 and suppose that the sensor gave this bba:
$m^\Omega(\{P\}) = 0.2$
$m^\Omega(\{M\}) = 0.4$
$m^\Omega(\{M \cup H\}) = 0.3$
$m^\Omega(\Omega) = 0.1$
After applying the pignistic transformation, we obtain:
$BetP\{P\} = 0.23$
$BetP\{M\} = 0.58$
$BetP\{H\} = 0.19$
We note that the most probable air target nature is Missile. Thus, if we have to decide, we will choose this hypothesis. ■

1.10 Conclusion

In this chapter, we have presented basic concepts of the theory of belief functions and we have exposed several tools proposed by this theory. This theory provides a flexible framework for handling knowledge of several levels of certainty going from total ignorance to perfect certainty.

This theory seems to be adapted to handle imperfect data and also to combine several knowledge provided by different sources in order to have only one with high level of certainty and more adapted to reality.

2

Discounting and reliability estimation in the theory of belief functions

2.1 Introduction

The theory of belief functions used for representing uncertainty is a strong tool for combination.
Indeed, this theory proposes a large number of combination rules to combine several evidential information. A problem may appear if sources of information are completely or even partially in conflict.

The conflict coming from the combination of conflicting evidential information incited the apparition of several methods intended to solve it. Some of these methods propose to solve the conflict when combining, like in (Dubois and Prade, 1988), (Murphy, 2000), (Smets and Kennes, 1994), (Martin and Osswald, 2007b) and (Yager, 1987), these combination rules hide the conflict regardless of its causes. Therefore, the conflict does not appear in the combined information because combination rules redistribute it with different manners.

Other methods consider that the main reason of the conflict apparition is the relative unreliability of at least one of the sources. Therefore, conflict solving can be insured by discounting the evidential information before combining proportionally to the source's degree of reliability but this method requires a preliminary knowledge of this degree of reliability.

In this chapter, we present methods which aim to solve conflict by discounting bbas before combining using their sources' degrees of reliability. These methods need an a

prior knowledge about sources' degrees of reliability which is not always supplied with bbas. Therefore source's reliability has to be learned or estimated.

In the second section of this chapter, we present some distance measures which are used in the theory of belief functions. These distances are used to estimate source's reliability in some methods, although other methods are based on reliability learning rather than estimating. In the third section of this chapter, we present some methods of source's reliability estimation and learning.
Source's reliability is used to discount the related bbas. Therefore, before concluding in the fifth section, we present in the fourth one some methods of discounting.

2.2 Distances between belief functions

The mass affected to the empty set when combining several beliefs is considered as an indicator about the conflict existing between their sources (Dempster, 1967), although the use of a distance can reflect better how far are bbas and also the degree of conflict between their sources.

2.2.1 Distance measures

In this section, some distances which can be used with belief functions are presented (Florea et al., 2009).

- **Euclidean distance (Cuzzolin, 2008):** Cuzzolin proposed an extension of the Euclidean distance from the probability theory to the theory of belief functions by replacing probability values by masses. The Euclidean distance between two bbas m_1^Ω and m_2^Ω supplied by s_1 and s_2 is defined as follows:

$$d(m_1^\Omega, m_2^\Omega) = \sqrt{\sum_{A \subseteq \Omega} [m_1^\Omega(A) - m_2^\Omega(A)]^2} \tag{2.1}$$

- **Bhattacharyya's distance:** Ristic and Smets (Ristic and Smets, 2006) proposed an extension of the Bhattacharyya's distance from probability theory to belief function theory as follows:

$$d(m_1^\Omega, m_2^\Omega) = \sqrt{1 - \sum_{A \subseteq \Omega} \sum_{B \subseteq \Omega} \sqrt{m_1^\Omega(A) m_2^\Omega(B)}} \tag{2.2}$$

  We notice that this distance is always null because $\sum_{A \subseteq \Omega} \sum_{B \subseteq \Omega} m_1^\Omega(A) m_2^\Omega(B)$ is always equal to 1, thus it has no use.

Therefore, Florea and Bossé (Florea et al., 2009) proposed a generalization of Bhattacharyya's distance extension to the theory of belief functions as follows:

$$d(m_1^\Omega, m_2^\Omega) = [\mid 1 - \sum_{A \subseteq \Omega} \sqrt{m_1^\Omega(A) m_2^\Omega(A)} \mid]^p \tag{2.3}$$

with p any positive number.

- **Jousselme distance (Jousselme et al., 2001):** This distance is specific to the theory of belief functions because of the matrix D which is defined on 2^Ω.
  This distance is balanced with Jaccard's coefficient $\frac{|A \cap B|}{|A \cup B|}$ as similarity measure between focal elements allowing the consideration of focal elements cardinality. Jousselme distance is defined as follows:

$$d(m_1^\Omega, m_2^\Omega) = \sqrt{\frac{1}{2}(m_1^\Omega - m_2^\Omega)^t \underline{\underline{D}}(m_1^\Omega - m_2^\Omega)} \tag{2.4}$$

  with :

$$D(A, B) = \begin{cases} 1 & \text{if A=B=}\emptyset \\ \frac{|A \cap B|}{|A \cup B|} & \forall A, B \in 2^\Omega \end{cases} \tag{2.5}$$

  We notice that Jousselme distance can be seen as an extension of Euclidean distance because this latter can be written as follows:

$$d(m_1^\Omega, m_2^\Omega) = \sqrt{(m_1^\Omega - m_2^\Omega)^t \underline{\underline{D}}(m_1^\Omega - m_2^\Omega)} \tag{2.6}$$

  with :

$$D(A, B) = \begin{cases} 1 & \text{if A=B} \\ 0 & \forall A, B \in 2^\Omega \end{cases} \tag{2.7}$$

  Other similarity functions (Florea et al., 2009) which may be used instead of Jaccard's coefficient are presented in table 2.1.

- **Tessem's distance (Tessem, 1993):** This distance is specific to the theory of belief functions because it takes the maximal distance between pignistic probabilities.

$$d(m_1^\Omega, m_2^\Omega) = max_{\omega \in \Omega} \mid BetP_1(\omega) - BetP_2(\omega) \mid \tag{2.8}$$

  This distance is computed from pignistic probabilities, therefore it does not take into account specificities of belief functions because two different belief functions with a low conflict may have the same pignistic probability function.

- **Ristic and Smets' dissimilarity measure (Ristic and Smets, 2006):** This dissimilarity measure is also specific to the theory of belief functions and it is defined as follows:

$$d(m_1^\Omega, m_2^\Omega) = -log(1 - m_{1 \bigcap 2}^\Omega(\emptyset)) \tag{2.9}$$

Table 2.1: Similarity functions

Similarity function's name	**Formula**
Dice	$\frac{2 \mid A \cap B \mid}{\mid A \mid + \mid B \mid}$
Sokal & Sneath 2	$\frac{\mid A \cap B \mid}{2 \mid A \cup B \mid - \mid A \cap B \mid}$
Kulczynski 2	$\frac{\mid A \cap B \mid}{2 \mid A \mid} + \frac{\mid A \cap B \mid}{2 \mid B \mid}$
Ochiai (used in (Fixsen and Mahler, 1997))	$\frac{\mid A \cap B \mid}{\sqrt{\mid A \mid \mid B \mid}}$

Unlike the distances previously described which take values in $[0, 1]$, values of this dissimilarity measure are in the interval $[0, +\infty]$.

Example 2.1 Suppose that two sources s_1 and s_2 provide us two bbas m_1^Ω and m_2^Ω.
$m_1^\Omega(P) = 0.5$, $m_1^\Omega(M) = 0.5$.
$m_2^\Omega(P) = 0.3$, $m_2^\Omega(H) = 0.3$, $m_2^\Omega(M) = 0.4$.
The distances between m_1^Ω and m_2^Ω are given in table 2.2.

Table 2.2: Distances between m_1^Ω and m_2^Ω

Distance name	**Distance value**
Euclidean distance (Cuzzolin, 2008)	0.37
Bhattacharyya's distance (Florea et al., 2009) ($p = 1$)	0.17
Jousselme distance (Jousselme et al., 2001)	0.26
Tessem's distance (Tessem, 1993)	0.3
Ristic and Smets' dissimilarity measure (Ristic and Smets, 2006)	1.0498

The extension of Bhattacharyya's distance proposed by Ristic and Smets is not used in table 2.2 because it has no use. ■

2.2.2 Discussion

Table 2.3 recapitulates hardness and weakness of each distance measure.

Table 2.3: Comparison of distance measures

Distance	**Hardness**	**Weakness**
Euclidean distance (Cuzzolin, 2008) and Bhattacharyya's distance (Florea et al., 2009)	Simplicity of computation.	It is extended from probability theory to belief function theory by replacing probabilities by masses; therefore it does not distinguish probabilities from masses (the same behavior).
Jousselme distance (Jousselme et al., 2001)	It takes into account specificities of belief functions by balancing the distance with focal elements cardinality and it is defined on 2^{Ω}.	Computation is not as simple as other distances.
Tessem's distance (Tessem, 1993)	Simplicity of computation.	Distance is computed from pignistic probabilities and not from masses.
Ristic and Smets' dissimilarity measure (Ristic and Smets, 2006)	Simplicity of computation.	Values of this dissimilarity measure are in the interval $[0, +\infty]$ rather than $[0, 1]$.

2.3 Reliability estimation in the theory of belief functions

Information about the reliability of the source is very useful because it indicates to the user the bba's degree of truth. In this section, we will present some methods of reliability estimation and learning.

2.3.1 Reliability estimation based on similarity matrix

The distance between two bbas is considered to be an indicator about their dissimilarity, and the complement of the dissimilarity to the unit is the similarity between bbas. Distances described in Section 2.2.1 may be used to measure the dissimilarity between bbas as well as those not quoted, but Jousselme distance may be the more appropriate one as it is the unique distance computed on masses with taking into account specificities of belief functions. Therefore, the similarity between two bbas is defined as follows:

$$S(m_i^\Omega, m_j^\Omega) = 1 - d_J(m_i^\Omega, m_j^\Omega) \tag{2.10}$$

with d_J is Jousselme distance. In general, the number of sources and provided bbas exceeds two, therefore the use of a similarity measure matrix (SMM) which resumes all similarity values between M sources providing each one a bba defined on the same frame of discernment and concerning the same problem. The SMM is defined as follows:

$$\text{SMM}(M) = \begin{bmatrix} 1 & S(m_1^\Omega, m_2^\Omega) & S(m_1^\Omega, m_3^\Omega) & \dots & S(m_1^\Omega, m_M^\Omega) \\ S(m_2^\Omega, m_1^\Omega) & 1 & S(m_2^\Omega, m_3^\Omega) & \dots & S(m_2^\Omega, m_M^\Omega) \\ \dots & \dots & \dots & \dots & \dots \\ S(m_M^\Omega, m_1^\Omega) & S(m_M^\Omega, m_2^\Omega) & S(m_M^\Omega, m_3^\Omega) & \dots & 1 \end{bmatrix} \tag{2.11}$$

Deng et al. (Deng et al., 2004) proposed to use the SMM to compute the support degree of a bba m_i^Ω as follows:

$$Sup(m_i^\Omega) = \sum_{1 \leq j \leq M \; j \neq i} \text{SMM}(i, j) \tag{2.12}$$

$Sup(m_i^\Omega)$ is the total similarity of m_i^Ω with all other bbas except itself. The normalization of $Sup(m_i^\Omega)$ defines the credibility degree of m_i^Ω which is defined as follows:

$$Crd(m_i^\Omega) = \frac{Sup(m_i^\Omega)}{\sum_{1 \leq j \leq M} Sup(m_j^\Omega)} \tag{2.13}$$

Guo et al. proposed in (Guo et al., 2006) an extension of (Deng et al., 2004) by defining the reliability of a source relatively to a bba.
A bba m_j^Ω is supposed to be fully reliable when its credibility ($Crd(m_j^\Omega)$) is the maximum, reliabilities of the other bbas are normalized proportionally to this maximum. Therefore, the reliability Rel of m_i^Ω is defined as follows:

$$Rel(m_i^\Omega) = \frac{Crd(m_i^\Omega)}{max_{1 \leq j \leq M} Crd(m_j^\Omega)} \tag{2.14}$$

For simplicity, equations (2.13) and (2.14) can be noted as follows:

$$Rel(m_i^\Omega) = \frac{Sup(m_i^\Omega)}{max_{1\leq j\leq M} Sup(m_j^\Omega)} \tag{2.15}$$

2.3.2 Reliability estimation based on conflict measure

Martin et al. proposed in (Martin et al., 2008) an other interpretation of the conflict, $k = m_\oplus^\Omega(\emptyset)$ is not used as a conflict measure between bbas. When combining a bba with it self, $k \neq 0$ is interpreted as the **auto-conflict** which reflects the intrinsic conflict of a bba. Therefore, the auto-conflict of order n is given by:

$$a = (\oplus_{i=1}^n m^\Omega)(\emptyset) \tag{2.16}$$

Due to the auto conflict, Martin et al. discard the relation between $m_\oplus^\Omega(\emptyset)$ and the conflict measure by proposing the use of a distance. The more far are two bbas, more they are conflicting, therefore they use Jousselme distance as a conflict measure (the choice of Joussselme distance is for the same reasons cited above).

In the general case, the number of sources and supplied bbas is great (M) and the distance measure is a binary one. Therefore, Martin et al. proposed two methods of conflict estimation based on distance measure:

- **Method 1:** The conflict of a bba m_i^Ω with other bbas is the average of Jousselme distances between m_i^Ω and each bba m_j^Ω with the constraint $i \neq j$:

$$Conf(i) = \frac{1}{M-1} \sum_{1\leq j\leq M \ j\neq i} d((m_i^\Omega, m_j^\Omega) \tag{2.17}$$

- **Method 2:** The conflict of m_i^Ω is the distance between m_i^Ω and the combined bba m_M^Ω result of the combination of all bbas except m_i^Ω using any combination rule.

$$Conf(i) = d(m_i^\Omega, m_M^\Omega) \tag{2.18}$$

Once the conflict $Conf(i)$ is computed, the source's relative reliability α_i according to the bba m_i^Ω is given by:

$$\alpha_i = [1 - Conf(i)^\lambda]^{1/\lambda} \tag{2.19}$$

with $\lambda > 0$ any real.

2.3.3 Reliability learning

Elouedi et al. in (Elouedi et al., 2004) proposed to find the reliability rate which minimizes the distance between the actual values of data and the pignistic probabilities computed from the discounted bbas.

In (Elouedi et al., 2004), authors consider a training set τ of n objects $\tau = \{o_1, \ldots, o_n\}$. Each object o_i belongs to a class c_j from the set of p classes $\Omega = \{\omega_1, \ldots, c_j, \ldots, \omega_p\}$.
For each object o_i, its class is known to be $c_j \in \Omega$ and this information is represented with an indicator factor $\sigma_{i,j}$ which is defined as follows:

$$\sigma_{i,j} = \begin{cases} 1 & \text{if } c_j = \omega_j \\ 0 & \text{otherwise} \end{cases} \tag{2.20}$$

A sensor supplies a bba $m^{\Omega}\{o_i\}$ defined on Ω representing its belief about the object o_i.
$m^{\Omega,\alpha}\{o_i\}$ is the discounted bba of $m^{\Omega}\{o_i\}$ and $BetP^{\alpha}$ is the pignistic probability computed from $m^{\Omega,\alpha}$ and representing the probability of the object o_i to belong to each class after discounting the related belief function.

To recapitulate, first steps of this method consist on computing pignistic probabilities from the discounted beliefs (with α an unknown parameter) and the indicator factor which reflects actual values of data.

In the next step, α is computed. α is the reliability rate which minimizes the total distance $TotalDist$ between $BetP^{\alpha}$ and the indicator function $\sigma_{i,j}$ using Euclidean distance:

$$TotalDist = \sum_{i=1}^{n}\sum_{j=1}^{p}(BetP^{\alpha}o_i(\omega_j) - \sigma_{i,j})^2 \tag{2.21}$$

When all bbas of τ are normalized, α can be found analytically as shown in (Elouedi et al., 2004). In this case, α is given by:

$$\alpha = min(1, max(0, \frac{\sum_{i=1}^{n}\sum_{j=1}^{p}(\sigma_{i,j} - BetP\{o_i\}(\omega_j))BetP\{o_i\}(\omega_j)}{n/p - \sum_{i=1}^{n}\sum_{j=1}^{p}BetP\{o_i\}(\omega_j)^2})) \tag{2.22}$$

Example 2.2 Table 2.4 is the example presented in (Elouedi et al., 2004), the frame of discernment is $\Omega = \{Airplane, Helicopter, Rocket\}$. In this example four bbas are

provided by s_1 representing classes of four objects. The total distance of s_1 ($TotalDist$)

Table 2.4: Example of reliability learning presented in (Elouedi et al., 2004)

s_1	o_1	o_2	o_3	o_4
$\emptyset$	0	0	0	0
$Airplane$	0	0	0	0
$Helicopter$	0	0.5	0.4	0
$Rocket$	0.5	0.2	0	0
$Airplane \cup Helicopter$	0	0	0	0
$Airplane \cup Rocket$	0	0	0.6	0.6
$Helicopter \cup Rocket$	0.3	0	0	0.4
Ω	0.2	0.3	0	0
real class	$Airplane$	$Helicopter$	$Airplane$	$Rocket$

is equal to:

$$TotalDist = \sum_{i=1}^{4}\sum_{j=1}^{3}(BetP^{\alpha_1}o_i(\omega_j) - \sigma_{i,j})^2.$$

$\alpha_1 = 0.66$ is the value of α which minimizes $TotalDist$. ■

2.3.4 Reliability estimation for mine detection

Milisavljević et al. considered in (Milisavjević et al., 2000) that a sensor works well in one scenario (when detecting especial types of soil, moisture...) and fails in another scenario, thus no single sensor in the field of humanitarian demining. Therefore, they used three sensors namely metal detector (MD), group penetrating radar (GPR) and infrared camera (IR). These sensors are complementary and are combined to improve detection results. Before combining, factors affecting the sensors' reliability have to be integrated in the related bbas

For short, we will describe the method presented in (Milisavjević, 2001) which considers only one sensor which is the infrared camera, the same process is applied to the other sensors before combining. In this method, an infrared (IR) camera considered as an IR sensor gives 2D horizontal images representing 2D projections of an object, these images are used to extract the surface, shape and size of an object.

A large number of mines has elliptical surface; namely mine of regular shape (MR), mine of irregular shape (MI), friendly (non-dangerous) object of regular shape (FR)

and friendly object of irregular shape (FI).
Therefore, the frame of discernment is $\Omega = \{MR,\ MI,\ FR,\ FI\}$.

Some measures are extracted from the detected shape and used to affect masses to subsets, these measures and the manner of affecting masses to subsets by the sensor are presented in table 2.5.

Table 2.5: Measures used to affect masses

Measures	**Focal elements**
Shape ellipticity	Masses are affected to $\{MR \cup FR\}$, $\{MI \cup FI\}$ and Ω.
Shape elongation	Masses are affected to $\{MR \cup FR\}$, $\{MI \cup FI\}$ and Ω.
Area\Size	Masses are affected according to a range of mine sizes: • If the size is within that rate, masses are assigned to Ω. • If it is out of that range, masses should be assigned to $\{FR \cup FI\}$.

The reliability of an IR sensor depends on:

- The quality of acquired data which affects the sensor when judging measures, when detecting the importance of each measure and its reliability.
- Particular weather conditions.
- Types of object under analysis.

Therefore, reliabilities [1] consist of three types of parameters defined for each source j and measure i, $i \in \{area\ a,\ elongation\ e,\ ellipse\ fitting\ f\}$:

- g_{ij}: is the confidence level of source j in its assessment when judging measure i. When $g_{ij} = 0$, the sensor is not reliable at all in its assessment and the source is fully reliable when $g_{ij} = 1$. g_{iI} is computed as follows:
  - **Area\size** g_{aI}: Area is moderately corrupted, it will not affect significantly mass assignment, it is not necessary to apply discounting on this measure.

[1] In (Milisavjević, 2001), reliability is called discounting factor

- **Ellipse fitting** g_{fI}: Ellipse fitting is computed from masses affected to $\{MR \cup FR\}$ and $\{MI \cup FI\}$ using shape ellipticity measure.

$$g_{fI} = \mid m_{fI}^{\Omega}(\{MR \cup FR\}) - m_{fI}^{\Omega}(\{MI \cup FI\}) \mid \tag{2.23}$$

- **Elongation** g_{eI}: Elongation is also computed from masses affected to $\{MR \cup FR\}$ and $\{MI \cup FI\}$ using shape elongation measure.

$$g_{eI} = \mid m_{eI}^{\Omega}(\{MR \cup FR\}) - m_{eI}^{\Omega}(\{MI \cup FI\}) \mid \tag{2.24}$$

- b_i: is the level of importance of measure i, $b_i \in [1, b_{scale}]$. There are several possibilities to estimate importance of measures:
  - The deminer determines how important is each measure to him.
  - Their values are fixed from the beginning.
  - They can be chosen from the beginning, initialized with the same value, and tuned after combination with regard to which subset has the highest mass.
- s_j: is deminer's confidence on sensor j, $s_j \in [1, s_{scale}]$. It depends on factors that can affect the reliability of each sensor such that environmental conditions.
  The deminer attributes for each sensor a number describing his belief in the reliability of this source.

When discounting, masses assigned by all measures i of a sensor j are discounted with g_{ij} and $\frac{b_j}{b_{scale}}$. After combining discounted masses provided by the same source, the combined masses (for all sources) are discounted with $\frac{s_j}{s_{scale}}$ and combined.

2.3.5 Discussion

Table 2.6 recapitulates hardness and weakness of each method of reliability estimation and learning.

2.4 Discounting

In some applications, it is possible to quantify source's degree of reliability reflecting its degree of expertise as shown in Section 2.3. When handling a bba, we have to take into account this degree of reliability because sources are not always fully reliable.

Taking into account the reliability of sources is named **discounting** as this operation weights beliefs by source's reliability degree. Thus, the degree of reliability of a source is taken into account by integrating it into all its bbas. Using discounting

operation in belief functions was first introduced by Shafer in 1976 (Shafer, 1976).

Discounting a bba m^{Ω} consists on weighting every mass $m^{\Omega}(B)$ by a coefficient $\alpha \in [0,1]$ called **reliability** and $(1-\alpha)$ is **the discount rate**. The discounted bba is given by:

$$\begin{cases} m^{\Omega,\alpha}(A) = \alpha \times m^{\Omega}(A) & \text{for } A \subset \Omega \\ m^{\Omega,\alpha}(\Omega) = (1-\alpha) + (\alpha \times m^{\Omega}(\Omega)) \end{cases} \tag{2.25}$$

Properties:

- **If** $\alpha = 1$**,** the source is fully reliable therefore discounting does not affect the bba:

$$\mathbf{m}^{\alpha}\mathbf{=m}$$

- **If** $\alpha = 0$**,** the source is fully unreliable therefore discounting induce to a vacuous belief function:

$$m^{\alpha} = m_0 \text{ with } m_0 \text{ is a VBF}$$

Example 2.3 Suppose that a sensor gives this bba:
$m^{\Omega}(P) = 0.5$
$m^{\Omega}(H) = 0.5$

Suppose that the degree of reliability of this sensor is equal to 0.3. The discounted bba is:
$m^{\Omega,\alpha}(P) = 0.5 \times 0.3 = 0.15$
$m^{\Omega,\alpha}(H) = 0.5 \times 0.3 = 0.15$
$m^{\Omega,\alpha}(\Omega) = 0.7 + 0 \times 0.3 = 0.7$ ■

The amount of belief due to the unreliability of the source is affected to the frame of discernment as total ignorance.

2.4.1 Plausibility discounting

The operator described above weakens or strengthens bbas, mass by mass, proportionally to source's reliability. Therefore, this operator does not affect focal elements but does change only masses.

Plausibility discounting proposed in (Zeng and Wu, 2007) is in three steps; in the first step plausibility function is computed from bbas as described in Chapter 1.

In the second step, plausibility functions are discounted using the following equation:

$$pl^{\alpha}(A) = [pl(A)]^{\alpha} \quad \forall A \subseteq \Omega \text{ and } A \neq \emptyset \tag{2.26}$$

Finally, discounted bbas are computed from discounted plausibility functions using the following equation:

$$\begin{cases} m^{\Omega,\alpha}(B) = \sum_{B \subseteq A} (-1)^{|A|-|B|+1} pl^{\alpha}(\bar{B}) \\ m^{\Omega,\alpha}(\emptyset) = 1 - pl^{\alpha}(\Omega) \end{cases} \tag{2.27}$$

Example 2.4 Suppose that we have a sensor namely s providing the following bba:
$m_s^{\Omega}(P) = 0.5$
$m_s^{\Omega}(H) = 0.5$

In the first step we compute the plausibility function corresponding to this bba using equation (1.16):
$pl(P) = 0.5$
$pl(M \cup P) = 0.5$
$pl(H) = 0.5$
$pl(M \cup H) = 0.5$
$pl(P \cup H) = 1$
$pl(\Omega) = 1$

Then, we discount this plausibility function using equation (2.26) with $\alpha = 0.3$:
$pl^{\alpha}(P) = 0.5^{0.3} = 0.8123$
$pl^{\alpha}(M \cup P) = 0.5^{0.3} = 0.8123$
$pl^{\alpha}(H) = 0.5^{0.3} = 0.8123$
$pl^{\alpha}(M \cup H) = 0.5^{0.3} = 0.8123$
$pl^{\alpha}(P \cup H) = 1^{0.3} = 1$
$pl^{\alpha}(\Omega) = 1^{0.3} = 1$

Finally, discounted bba is computed from the discounted plausibility function using equation (2.27):
$m_s^{\Omega,\alpha}(P) = 0.1877$
$m_s^{\Omega,\alpha}(H) = 0.1877$
$m_s^{\Omega,\alpha}(P \cup H) = 0.6246$

The initial bba has only two focal elements which are P and H but after discounting using plausibility discounting operator the number of focal elements is up to three which are P, H and $\{P \cup H\}$.

We notice also that the amount of belief due the unreliability of the source is not necessarily affected to the total ignorance when the plausibility discounting operator is used.

In example 2.3, the same bba was discounted using the mass discounting operator. The amount of total ignorance in the discounted bba is about 0.7 but the amount of total ignorance in the discounted bba using plausibility discounting is kept null. Thus, plausibility discounting avoids loosing information more than mass discounting. ■

2.4.2 Sequential discounting

A method of sequential discounting is presented by Schubert in (Schubert, 2008).

In (Schubert, 2008), a high degree of conflict is seen as if combined belief functions are not referring to the same problem (may be referring to sub problems that should not be combined) or they are expressed on different frames of discernment, therefore at least one piece of evidence does not respect the representation of the frame of discernment. A small quantity of conflict is seen as an error of measurement.

The mass affected to the empty set after combining several belief functions using Dempster's rule of combination is considered as the amount of conflict. Therefore, if S sources provide S bbas (a bba for each source), the conflict c is the mass affected to the empty set when combining these S bbas.

$$c = \oplus m_S^{\Omega}(\emptyset) \tag{2.28}$$

In this method, Schubert distinguishes two measures of conflict. The first one is the conflict computed using equation (2.28), c is the conflict involving all bbas.
The second measure of conflict is computed by supposing that a given bba m_j^{Ω} with $(j \leq S)$ does not belong to the set of bbas to be combined, in other words a conflict c^j is computed by combining the existing S bbas except the jth one (m_j^{Ω}).

The decrease of conflict from c to c^j reflects the contribution of the jth bba on the conflict, and the normalization of the conflict's drop is considered to be the **degree of falsity** of that belief function m_j^{Ω}.
The degree of falsity of a bba j is computed as follows:

$$f_j = \frac{c - c^j}{1 - c^j} \tag{2.29}$$

The method of sequential discounting consists on fixing a maximal level of tolerable conflict c_{max} from the beginning, then discounting each bba proportionally to its degree

of falsity in sequential steps until having a conflict which is lower or equal to fixed level. In a step d of sequential discounting the degree of falsity of a bba $m^{\Omega,d}$ is computed from $m^{\Omega,d-1}$ (the bba out put of the $d-1$ step). This falsity degree is used to discount $m^{\Omega,d-1}$ to have $m^{\Omega,d}$.
The discounting operator used in this method is given by:

$$\begin{aligned} m^{\Omega,d}(A) &= 1 - [\epsilon(\frac{c - c^j}{1 - c^j})]m^{\Omega,d-1} \quad \forall A \in \Omega \\ m^{\Omega,d}(\Omega) &= 1 - \sum_{A \subset \Omega} m^{\Omega,d}(A) \end{aligned} \tag{2.30}$$

with $\epsilon < 1$ a gain factor.

After fixing c_{max} the maximum level of tolerable conflict and ϵ the gain factor, the sequential discounting is in four steps which are repeated many times:

- Step 1: In this step c is computed using equation (2.28). If $c \leq c_{max}$ the process is finished else steps 2, 3 and 4 are processed.
- Step 2: c_j is computed for each m_j^{Ω} $(1 \leq j \leq S)$.
- Step 3: f_j is computed for each bba using equation (2.29).
- Step 4: discount each bba (output of the previous iteration) with its degree of falsity f to obtain a discounted bba which may be used in the next iteration.

Example 2.5 Suppose that we have three bbas provided by different sources:
The bba provided by s_1 is: $m_{s_1}^{\Omega}(P) = 0.5$, $m_{s_1}^{\Omega}(H) = 0.5$.
The bba provided by s_2 is: $m_{s_2}^{\Omega}(P) = 0.5$, $m_{s_2}^{\Omega}(M) = 0.5$.
The bba provided by s_3 is: $m_{s_3}^{\Omega}(P) = 0.3$, $m_{s_3}^{\Omega}(H) = 0.3$, $m_{s_3}^{\Omega}(M) = 0.4$.
Table 2.7 describes steps of sequential discounting performed on these bbas using $c_{max} = 0.8$ and $\epsilon = 0.1$.
In table 2.7, values are rounded off but in the calculation they are not.

Sequential discounting process is stopped at the fourth iteration because $c < c_{max}$ and:
The discounted bba of s_1 is: $m_{s_1}^{\Omega,\alpha}(P) = 0.4$, $m_{s_1}^{\Omega,\alpha}(H) = 0.4$, $m_{s_1}^{\Omega,\alpha}(\Omega) = 0.2$.
The discounted bba of s_2 is: $m_{s_2}^{\Omega,\alpha}(P) = 0.4$, $m_{s_2}^{\Omega,\alpha}(M) = 0.4$, $m_{s_2}^{\Omega,\alpha}(\Omega) = 0.2$.
The discounted bba of s_3 is: $m_{s_3}^{\Omega,\alpha}(P) = 0.25$, $m_{s_3}^{\Omega,\alpha}(H) = 0.25$, $m_{s_3}^{\Omega,\alpha}(M) = 0.32$, $m_{s_3}^{\Omega,\alpha}(\Omega) = 0.18$. ■

The use of this method allows choosing the degree of maximal tolerable conflict, there will be many iterations to be performed (big time of computation). Also, the mass affected to the empty set is not really the conflict value because it involves a degree of auto-conflict due to the non idempotency of the rule.

2.4.3 Contextual discounting

The contextual discounting is proposed by Mercier et al. in (Mercier et al., 2005) and the main idea is that the reliability of a source may be dependent on the variable of interest's real value, like in medicine area, a doctor can be particularly competent to diagnosis a particular disease more than others due to his past experience or training. Also, the reliability of a sensor may be dependent on the weather, its background and target properties, therefore its reliability can be dependent on the real target.

Using contextual discounting, source's reliability is defined on different contexts conditionally on different variable's hypotheses. Therefore, a source have different values of reliability depending on the hypotheses of Ω. Thus, the reliability of a source is defined conditionally on each $\omega_k \in \Omega$ with $k = 1 \ldots, K$.

The classical discounting operator is characterized by a reliability value α and the discounted bba is noted $m^{\Omega,\alpha}$ but the contextual discounting is characterized by a vector of reliability values $(\alpha_1, \ldots, \alpha_K)$, each reliability value α_k is associated to an hypothesis ω_k.
The contextual discounted bba is noted $m^{\Omega,(\alpha)}$ and computed as follows:

$$m^{\Omega,(\alpha)}(A) = \sum_{B \cup C = A} [\prod_{\omega_i \in c} (1 - \alpha_i) \times \prod_{\omega_j \in \bar{c}} \alpha_j] m^{\Omega}(B) \qquad \forall A \subseteq \Omega \tag{2.31}$$

Equation (2.31) is equivalent to the following one:

$$m^{\Omega,(\alpha)}(A) = \sum_{B \subseteq A} G(A, B) \times m^{\Omega}(B) \qquad \forall A \subseteq \Omega \tag{2.32}$$

with:

$$G(A, B) = \prod_{\omega_i \in A \setminus B} (1 - \alpha_i) \prod_{\omega_j \in \bar{A}} \alpha_j \qquad \forall B \subseteq 1 \subseteq \Omega \tag{2.33}$$

$G(A, B)$ is a generalization matrix (Smets, 2002).

Contextual discounting operator is not a generalization of the classical discounting operator. Therefore, discounting a bba with its source's reliability α using classical discounting operator is different from discounting the same bba using contextual discounting operator with source's reliability vector even if $\alpha_i = \alpha$, $1 \leq i \leq K$.

Example 2.6 An example presented in (Mercier et al., 2005) supposes that a bba m^{Ω} is defined on a frame of discernment $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and provided by a source having a reliability vector $(\alpha) = (\alpha_1, \alpha_2, \alpha_3)$, the discounted bba $m^{\Omega,(\alpha)}$ is also given in (Mercier et al., 2005):

$m^{\Omega,(\alpha)}(\emptyset) = \alpha_1\ \alpha_2\ \alpha_3\ m^{\Omega}(\emptyset)$
$m^{\Omega,(\alpha)}(\omega_1) = \alpha_2\ \alpha_3\ [m^{\Omega}(\omega_1) + \alpha_1\ m^{\Omega}(\emptyset)]$
$m^{\Omega,(\alpha)}(\omega_2) = \alpha_1\ \alpha_3\ [m^{\Omega}(\omega_2) + \alpha_2\ m^{\Omega}(\emptyset)]$
$m^{\Omega,(\alpha)}(\omega_3) = \alpha_1\ \alpha_2\ [m^{\Omega}(\omega_3) + \alpha_3\ m^{\Omega}(\emptyset)]$
$m^{\Omega,(\alpha)}(\omega_1 \cup \omega_2) = \alpha_3\ [m^{\Omega}(\omega_1 \cup \omega_2) + (1-\alpha_1)\ m^{\Omega}(\omega_2) + (1-\alpha_2)\ m^{\Omega}(\omega_1) + (1-\alpha_1)\ (1-\alpha_2)\ m^{\Omega}(\emptyset)]$
$m^{\Omega,(\alpha)}(\omega_1 \cup \omega_3) = \alpha_2\ [m^{\Omega}(\omega_1 \cup \omega_3) + (1-\alpha_1)\ m^{\Omega}(\omega_3) + (1-\alpha_3)\ m^{\Omega}(\omega_1) + (1-\alpha_1)\ (1-\alpha_3)\ m^{\Omega}(\emptyset)]$
$m^{\Omega,(\alpha)}(\omega_2 \cup \omega_3) = \alpha_1\ [m^{\Omega}(\omega_2 \cup \omega_3) + (1-\alpha_2)\ m^{\Omega}(\omega_3) + (1-\alpha_3)\ m^{\Omega}(\omega_2) + (1-\alpha_2)\ (1-\alpha_3)\ m^{\Omega}(\emptyset)]$
$m^{\Omega,(\alpha)}(\Omega) = m^{\Omega}(\Omega) + (1-\alpha_1)\ m^{\Omega}(\omega_2 \cup \omega_3) + (1-\alpha_2)\ m^{\Omega}(\omega_1 \cup \omega_3) + (1-\alpha_3)\ m^{\Omega}(\omega_1 \cup \omega_2) + (1-\alpha_1)\ (1-\alpha_2)\ m^{\Omega}(\omega_3) + (1-\alpha_2)\ (1-\alpha_3)\ m^{\Omega}(\omega_1) + (1-\alpha_1)\ (1-\alpha_3)\ m^{\Omega}(\omega_2) + (1-\alpha_1)\ (1-\alpha_2)\ (1-\alpha_3)\ m^{\Omega}(\emptyset)$

Suppose that we have the following bba m^{Ω} defined on $\Omega = \{M,\ H,\ P\}$:
$m^{\Omega}(H) = 0.3$
$m^{\Omega}(P) = 0.2$
$m^{\Omega}(H \cup P) = 0.4$
$m^{\Omega}(\Omega) = 0.1$
This bba is provided by a source having a reliability vector (0.2, 0.7, 0.9). The discounted bba using contextual discounting is:
$m^{\Omega,(\alpha)}(H) = 0.189$
$m^{\Omega,(\alpha)}(P) = 0.036$
$m^{\Omega,(\alpha)}(H \cup P) = 0.585$
$m^{\Omega,(\alpha)}(H \cup M) = 0.021$
$m^{\Omega,(\alpha)}(M \cup P) = 0.004$
$m^{\Omega,(\alpha)}(\Omega) = 0.165$

■

2.4.4 Discussion

Table 2.8 recapitulates specificities of each method of discounting.

2.5 Conclusion

In this chapter, we have presented some distance measures which are applied in the theory of belief functions. Jousselme distance is the most adapted distance to belief functions thanks to its characteristics which agree with belief functions' specificities.

We have also presented two methods which estimate the reliability of a source using Jousselme distance and an other method which learns the reliability using a training set.

Once source's reliability is estimated or learned, discounting its bba allows taking into account this reliability degree on provided information and also reduces the conflict which may appear when combining these bbas with other ones supplied by other sources.

In Chapter 3, we will present a generalization of Martin et al. (Martin et al., 2008) method for reliability estimation to be adequate with evidential databases. Evidential databases store a great number of bbas supplied by the same source and all these bbas have to be involved when estimating their source's reliability.

Table 2.6: Comparison of methods of reliability estimation and learning

Method	**Hardness**	**Weakness**
Reliability estimation based on similarity matrix	The use of a distance as a dissimilarity measure, more bbas are far more they are dissimilar.	In this method, a bba is supposed to be fully reliable when its credibility is the maximum of all credibility values computed for all bbas provided by the same source. In practice, a bba may have the maximal credibility without being fully reliable. Bbas provided by an unreliable source are unreliable even the bba having the maximal credibility. This method estimates the reliability of only one bba per source.
Reliability estimation based on conflict measure	The use of distance measure as an indicator about the degree of conflict between bbas as well as their sources discards the amount of auto-conflict. The reliability of a source depends on its conflict with all other sources.	This method considers only one bba when estimating source's reliability.
Reliability learning	Source's reliability is learned from all bbas.	This method is used to learn sources' reliability rate in the purpose of improving their classification results but the used discounting operator does not improve classification results. $TotalDist$ is a scalar constrained nonlinear programming problem.
Reliability estimation for mine detection	This method takes into account all the factors which affect sensor's reliability.	The deminer provides b_i and s_j, therefore without deminer discounting operation cannot be performed. Factors may be numerous and it is not always possible to quantify them. This method is not applicable to all fields.

Table 2.7: Sequential discounting

Iteration	**bbas**	**c**	$\mathbf{c_j}$	**f**	**Discounted bbas**
	$P(0.5)\ H(0.5)$		0.65	0.79	$P(0.46)\ H(0.46)\ \Omega(0.08)$
1	$P(0.5)\ M(0.5)$	0.925	0.7	0.75	$P(0.46)\ M(0.46)\ \Omega(0.08)$
	$P(0.3)\ H(0.3)\ M(0.4)$		0.75	0.7	$P(0.28)\ H(0.28)\ M(0.37)\ \Omega(0.07)$
	$P(0.46)\ H(0.46)\ \Omega(0.08)$		0.56	0.698	$P(0.43)\ H(0.43)\ \Omega(0.14)$
2	$P(0.46)\ M(0.46)\ \Omega(0.08)$	0.867	0.6	0.67	$P(0.43)\ M(0.43)\ \Omega(0.14)$
	$P(0.28)\ H(0.28)\ M(0.37)\ \Omega(0.07)$		0.64	0.63	$P(0.26)\ H(0.26)\ M(0.35)\ \Omega(0.13)$
	$P(0.43)\ H(0.43)\ \Omega(0.14)$		0.49	0.63	$P(0.4)\ H(0.4)\ \Omega(0.2)$
3	$P(0.43)\ M(0.43)\ \Omega(0.14)$	0.81	0.52	0.6	$P(0.4)\ M(0.4)\ \Omega(0.2)$
	$P(0.26)\ H(0.26)\ M(0.35)\ \Omega(0.13)$		0.56	0.58	$P(0.25)\ H(0.25)\ M(0.32)\ \Omega(0.18)$
	$P(0.4)\ H(0.4)\ \Omega(0.2)$		-	-	-
4	$P(0.4)\ M(0.4)\ \Omega(0.2)$	0.69	-	-	-
	$P(0.25)\ H(0.25)\ M(0.32)\ \Omega(0.18)$		-	-	-

Table 2.8: Comparison of discounting methods

Discounting method	**Hardness**	**Weakness**
Classical discounting method (Shafer, 1976)	Discounts masses proportionally to source's reliability degree and affect the discount rate due to source's unreliability to Ω.	This method does not affect the set of focal elements, but a source may make mistakes when affecting masses and also when choosing focal elements. Mistakes made when choosing focal elements are not considered.
Plausibility discounting (Zeng and Wu, 2007)	Takes into account mistakes made when choosing focal elements, this method changes the set of focal elements and the discount rate is not totally affected to Ω.	This method does not discount bbas directly but bbas are discounted using their plausibility functions.
Sequential discounting (Schubert, 2008)	The maximum tolerable conflict is fixed. After discounting the conflict is lower than this threshold.	This method needs many iterations especially if the number of sources and bbas is great.
Contextual discounting (Mercier et al., 2005)	Takes into account the background, training and expertise of the source according to each hypothesis of Ω.	Most of the time source's reliability is not known, for this method a vector of reliabilities has to be learned or estimated and not only an unique value.

3

Reliability estimation in evidential databases

3.1 Introduction

These last years are marked by the emergence of information in different fields, these information are scattered and not exploited. Grouping information concerning the same field gives a high quantity of data to be stored and information become more and more bulky therefore the use of databases is a necessity.

As these databases integrate a high quantity of information provided by different sources, these data may be conflicting because their sources may disagree.

Most of researches are based on perfect and certain data but data stored in databases or data warehouses are most of the time imperfect (incomplete or uncertain) as well as conflicting and users need these data for decision making. Therefore, imperfect data have to be exploited as well as those perfect and the conflict has to be solved.

The number of evidential databases is high in some fields and analyzing all of these databases takes a lot of time and energy, thus the user have to take into account all these evidential databases to make the right decision and to take into account different experts' points of view. Integrating these databases will reduce the quantity of data to be stored and handled by the user.

This chapter is organized as follows: in the second section, we present a definition of an evidential database as presented in (Bach Tobji et al., 2008). Next, we detail our approach of reliability estimation from evidential databases as we have presented in (Chebbah et al., 2010b) and (Chebbah et al., 2010a). Our approach is a generalization of Martin et al.'s method (Martin et al., 2008) presented in the previous chapter. Our

method consists on estimating source's reliability on the bases of all its bbas stored in its evidential database. These reliabilities will be used in the last section of this chapter to discount all bbas stored in evidential databases before integrating them.

3.2 Evidential databases

An **evidential database** (*EDB*), also called **DS database**, is a database containing certain and/or uncertain data, uncertainty is expressed using the theory of belief functions as presented in (Bach Tobji et al., 2008) and (Hewawasam et al., 2005).

An evidential database is a database having X records and Y attributes such that each attribute y ($1 \leq y \leq Y$) has a domain Ω_y containing all its possible values. Ω_y is the *frame of discernment* of the yth attribute (Hewawasam et al., 2005).

An EDB must have at least one **evidential attribute**, values of this attribute are uncertain and are represented with different bbas as defined in (Bach Tobji et al., 2008). An **evidential value** V_{xy} for the xth record and the yth attribute is a bba such that:

$$\begin{array}{l} m_{xy}^{\Omega_y} : 2^{\Omega_y} \rightarrow [0,1] \text{ with :} \\ m_{xy}^{\Omega_y}(\emptyset)=0 \text{ and } \sum_{A \subseteq D_y} m_{xy}^{\Omega_y}(\text{A})=1 \end{array} \tag{3.1}$$

Example 3.1 An example of an evidential database is described in table 3.1, this evidential database contains target detected by several sensors. The attribute target is the only evidential attribute in this evidential database, its frame of discernment is $\Omega_{target} = \{Plane\ P,\ Helicopter\ H,\ Missile\ M\}$.

Table 3.1: Example of an EDB

id	**Sensor**	**Time**	**Target**
1	s_1	t_1	$P(0.2)$ $\{P \cup H\}(0.6)$ $\{P \cup H \cup M\}(0.2)$
2	s_2	t_1	$P(0.5)$ $H(0.5)$
3	s_3	t_1	$P(0.5)$ $M(0.2)$ $\{P \cup M\}(0.3)$

■

This evidential database stores data of different levels of certainty. It stores:

- **Perfect information:** The focal element is a singleton having a mass equal to one. The value of the evidential attribute is a certain belief function.
- **Probabilistic information:** The value of the evidential attribute is a bba with several focal elements which are singletons. Line 2 of table 3.1 is an example of probabilistic information.
- **Possibilistic information:** Line 1 of table 3.1 is a possibilistic information as focal elements are nested.
- **Missing information:** Missing information is represented by a bba with only one focal element which is the frame of discernment Ω, the corresponding bba is a vacuous belief function.
- **Evidential information:** When information is neither perfect, nor probabilistic, nor possibilistic nor missing then it is evidential. The bba defined for the attribute target in line 3 of table 3.1 is an evidential information.

Evidential databases are used in different areas such that classification where they stock bbas supplied by different classifiers such as in (Hewawasam et al., 2005).

Sources of information stored in EDBs are numerous and the quantity of stored information is high, although they report about the same thing most of the time. Fusion of these information decreases their quantity and helps users for decision making but the main problem is the conflict which may appear while confronting several information reflecting different experts' opinions. We will use the theory of evidence (the theory of belief functions) for combining information provided by different sources.

3.3 Reliability estimation based on conflict estimation

An evidential database is used to store different bbas supplied by a source, therefore the number of evidential databases is dependent on the number of sources. Having S sources implies the existence of S evidential databases such that every EDB belongs to an unique source.

Also, having a high number of EDBs implies that the quantity of data to be stored is high and these data may some times represent the same piece of information.
Integrating these S evidential databases reduces the quantity of data to be stored and

also helps the user in decision making, thus the latter has to take into account only one integrated EDB which resumes S ones.

When integrating evidential values from several EDBs, a conflict may appear due to the variety of EDBs' sources which might be conflicting.

Example 3.2 Table 3.2 and table 3.3 are two tables of two different EDBs to integrate, these EDBs are supplied by different sources namely s_1 and s_2. Integrating these tables consists on combining the stored bbas, values of the attribute target corresponding to the same object.

We note that in this chapter, we assume that a sensor detects a target only once. Thus, targets detected by a source (sensor) s at t_1 and t_2 are different.

Table 3.2: EDB of s_1

id	**Time**	**Target**
1	t_1	P
2	t_2	$P(0.5)$ $H(0.5)$

Table 3.3: EDB of s_2

id	**Time**	**Target**
1	t_1	P
2	t_2	$P(0.2)$ $H(0.6)$ $\{P \cup H\}(0.2)$

To integrate these two tables, values of the attribute target have to be combined. Therefore the bbas' values of the attribute target for the first object in both tables (line 1 in EDB of s_1 and line 1 in EDB of s_2) have to be combined; these two bbas are certain belief functions which are not contradictory. Combining these two values does not raise any problem contrary to second values of this attribute for both EDBs because a non null mass is affected to the $\emptyset$ alarming about the conflict (Dempster, 1967) although $m^{\Omega}(\emptyset)$ includes an amount of auto-conflict due to the non idempotency of the combination rule (Martin et al., 2008) and (Osswald and Martin, 2006).

Table 3.4 is the integrated table of table 3.2 and table 3.3, the conflict is marked by a non null mass affected to the $\emptyset$.

Table 3.4: EDB of s_2

id	Time	Target
1	t_1	P
2	t_2	$P(0,2)$ $H(0.4)$ $\emptyset(0.4)$

■

Different manners of conflict solving are presented in Chapter 1, one manner is to prevent the conflict before it happens by discounting bbas using the source's degree of reliability. This method takes into account the degree of reliability of each source before combining their bbas, thus the conflict is eliminated or reduced from the beginning and even if it appears after the combination it will be bearable.

In this chapter, we suggest a method which aims to solve the conflict appearing when combining several bbas while integrating tables from several EDBs.

This method is a generalization of (Martin et al., 2008) and it is in three steps; in the first step we compute the conflict of a source against all other existing sources as described in Section 3.3.1. Then, the reliability of this source is estimated on the bases of its conflict values as described in Section 3.3.3 and finally, all data are discounted proportionally to their source's reliability degree.

3.3.1 Conflict estimation

Martin et al. (Martin et al., 2008) consider that more a bba is distant from an other one, more they are conflicting. Thus, the distance between two bbas reflects the degree of conflict between them and also the degree of conflict between their sources.

Jousselme distance (Jousselme et al., 2001) is used because it takes into account specificities of belief functions (the matrix D is defined on the set of all subsets of Ω namely 2^Ω) but distances present in Chapter 2 can also be used.

$$d(m_1^\Omega, m_2^\Omega) = \sqrt{\frac{1}{2}(m_1^\Omega - m_2^\Omega)^t \underline{\underline{D}}(m_1^\Omega - m_2^\Omega)} \tag{3.2}$$

with :

$$D(A,B) = \begin{cases} 1 & \text{if A=B=}\emptyset \\ \frac{|A \cap B|}{|A \cup B|} & \forall A, B \in 2^{\Omega} \end{cases} \tag{3.3}$$

The degree of conflict between two sources (s_1 and s_2) is the distance between their corresponding bbas, respectively m_1^{Ω} and m_2^{Ω}.

$$Conf(s_1, s_2) = d(m_1^{\Omega}, m_2^{\Omega}) \tag{3.4}$$

Example 3.3 Let us take the same example 3.2, note that $m_{s_i j}^{\Omega}$ is the jth bba value of the attribute target (corresponds to the jth record) for the ith source (s_i).

Table 3.5: EDB of s_1

id	Time	Target
1	t_1	$m_{s_1 1}^{\Omega}$: P
2	t_2	$m_{s_1 2}^{\Omega}$: $P(0.5)$ $H(0.5)$

Table 3.6: EDB of s_2

id	Time	Target
1	t_1	$m_{s_2 1}^{\Omega}$: P
2	t_2	$m_{s_2 2}^{\Omega}$: $P(0.2)$ $H(0.6)$ $\{P \cup H\}(0.2)$

$d(m_{s_1 1}^{\Omega}, m_{s_2 1}^{\Omega}) = 0$ then $Conf(s_1, s_2) = 0$. For this first record, both sources agree on the value of the attribute target.
$d(m_{s_1 2}^{\Omega}, m_{s_2 2}^{\Omega}) = 0.2236$ then $Conf(s_1, s_2) = 0.2236$. For this second record, both sources disagree partially on the value of the attribute target and the degree of conflict is about 0.2236.

■

This distance measure is a binary measure because it computes the distance between only two bbas reflecting the conflict between their sources with a restriction on the number of sources which has to be equal to 2.
When the number of sources exceeds 2 ($S > 2$), the conflict of a given source s_j can be computed in two different ways, each way is a different type of distance, therefore we distinguish two types of distance.

- **Distance type 1:** is the mean of distances between a bba m_j^Ω supplied by the source s_j and other bbas without using a combination rule.
  For S sources, the conflict of the source s_j is the mean of distances between m_j^Ω, the bba supplied by s_j and all the other $S-1$ bbas one by one. For each bba m_i^Ω from $S-1$ bbas (all the bbas except m_j^Ω supplied by s_j, the source subject of the conflict estimation), the distances between m_j^Ω and each m_i^Ω are computed. Therefore $S-1$ values of distance are obtained. The conflict of s_j is the mean of these $S-1$ distances.

$$d(m_j^\Omega, m_M^\Omega) = \frac{1}{S-1} \times \sum_{i=1, i\neq j}^{S-1} d(m_j^\Omega, m_i^\Omega) \tag{3.5}$$

Example 3.4 Suppose that we have three sources s_1, s_2 and s_3 supplying respectively $m_{s_1}^\Omega$, $m_{s_2}^\Omega$, and $m_{s_3}^\Omega$: ■

Table 3.7: Distance type 1

Source	**Bba**	**Distance type 1**	**Conflict**
s_1	$m_{s_1}^\Omega$: $P(0.5)$ $\{P \cup H \cup M\}(0.5)$	$d(m_{s_1}^\Omega, m_{s_2}^\Omega) = 0.5323$ $d(m_{s_1}^\Omega, m_{s_3}^\Omega) = 0.6164$	0.57435
s_2	$m_{s_2}^\Omega$: $P(0.2)$ $H(0.6)$ $\{P \cup H\}(0.2)$	$d(m_{s_2}^\Omega, m_{s_1}^\Omega) = 0.5323$ $d(m_{s_2}^\Omega, m_{s_3}^\Omega) = 0.7832$	0.65775
s_3	$m_{s_3}^\Omega$: $M(0.6)$ $\{P \cup M\}(0.4)$	$d(m_{s_3}^\Omega, m_{s_1}^\Omega) = 0.6164$ $d(m_{s_3}^\Omega, m_{s_1}^\Omega) = 0.7832$	0.6998

- **Distance type 2:** is the distance between a bba m_j^Ω supplied by the source s_j and the combined bba of all other bbas except m_j^Ω. This method needs the use of a combination rule to combine the $S-1$ bbas. Combination rules described in Chapter 1 may be used in this context as well as those not quoted.

  For S sources, the conflict of the source s_j with all the other sources corresponds to the distance between m_j^Ω, the bba supplied by this source, and m_M^Ω representing the combined bba of the $S-1$ bbas supplied by all the other sources.

  Example 3.5 Let us continue with the same example 3.4 using Dempster's rule of combination. The conflict of each source using distance type 2 is given in table 3.8: ■

Table 3.8: Distance type 2

Source	Bba	Distance type 2	Conflict
s_1	$m^{\Omega}_{s_1}$: $P(0.5)$ $\{P \cup H \cup M\}(0.5)$	$d(m^{\Omega}_{s_1}, m_M) = 0.4082$	0.4082
s_2	$m^{\Omega}_{s_2}$: $P(0.2)$ $H(0.6)$ $\{P \cup H\}(0.2)$	$d(m^{\Omega}_{s_2}, m^{\Omega}_M) = 0.6650$	0.6650
s_3	$m^{\Omega}_{s_3}$: $M(0.6)$ $\{P \cup M\}(0.4)$	$d(m^{\Omega}_{s_3}, m^{\Omega}_M) = 0.7282$	0.7282

Remarque: Temporal complexity of distance type 2 is lower than temporal complexity of distance type 1 especially if the number of bbas is great. This difference in temporal complexities is due to the number of uses of Jousselme distance which is great in distance type 1 (proportionally to the number of bbas) and low in distance type 2 (it is used only once).

3.3.2 Source's relative reliability estimation

Once source's degree of conflict according to a bba is computed, the **relative reliability** of this source can be also computed. Martin et al. in (Martin et al., 2008) proposed a method to estimate the relative reliability α_j of a source s_j based on the conflict measure as follows:

$$\alpha_j = (1 - Conf(s_j, s)^{\lambda})^{\frac{1}{\lambda}} \tag{3.6}$$

λ is a not null real.

Example 3.6 Once conflict is estimated for each source in example 3.5, reliability estimation is done using equation (3.6) for $\lambda = \frac{1}{2}$ as described is table 3.9. ■

Table 3.9: Reliability estimation

Source	Conflict	Reliability
s_1	0.4082	0.1304
s_2	0.6650	0.034
s_3	0.7282	0.0215

3.3.3 Source's global reliability estimation

The coefficient α_j is called relative reliability because it takes into account only one bba. In practice, an evidential database stocks a greet number of bbas supplied by the same source. Source's reliability has to take into account all bbas supplied by this source thus it is computed from all its relative reliabilities.

For example, let S be the number of EDBs corresponding to S sources. Every EDB has X records and Y attributes, thus each source from S ones has $X \times Y$ relative reliabilities.

In this context, we suggest to use the mean of $X \times Y$ relative reliabilities as the **source's global reliability**. The mean of relative reliabilities is chosen because a source keeps, in general, the same level of reliability. Although a source may sometimes make a mistake and be reliable or unreliable while it is not in general, it keeps in average the same level of reliability.

Choosing the mean avoids using extreme values like minimum and maximum for discounting. Indeed, discounting using the minimal reliability reduces bbas to the total ignorance and discounting using the maximal reliability keeps bbas unchanged, but discounting using the mean improves source's reliability and keeps bbas' integrity.

Therefore, the global reliability a_j^g of a source s_j is the mean of its X relative reliabilities α_{xj}:

$$\alpha_j^g = \frac{1}{X}\sum_{x=1}^{X}(\alpha_{xj}) \tag{3.7}$$

Example 3.7 Let us take the same example 3.3 with three sources s_1, s_2 and s_3, their bbas are given in tables 3.10, 3.11 and 3.12.

Table 3.10: EDB of s_1

id	Time	Target
1	t_1	P
2	t_2	$P(0.5)$ $H(0.5)$

The conflict and reliabilities for both s_1, s_2 and s_3 using distance type 1 and $\lambda = \frac{1}{2}$ are obtained using equations (3.5) and (3.6). Global reliabilities computation is described in table 3.13. ■

Table 3.11: EDB of s_2

id	Time	Target
1	t_1	P
2	t_2	$P(0.2)$ $H(0.6)$ $\{P \cup H\}(0.2)$

Table 3.12: EDB of s_3

id	Time	Target
1	t_1	M
2	t_2	$M(0.6)$ $\{P \cup H\}(0.4)$

3.3.4 Combination's reliability estimation

Let S be the number of sources supplying, each one, a bba. For each bba, a relative reliability α_j is computed by estimating the conflict between its source s_j $(1 \leq j \leq S)$ and all the others using a distance measure.

A reliability degree can be affected to the bba result of combining these S bbas in order to indicate to the user how much the combined information used for decision making is reliable.

Combination's reliability α_c is the mean of S combined bbas' relative reliabilities α_j.

$$\alpha_c = \frac{1}{S} \times \sum_{j=1}^{S} (\alpha_j) \tag{3.8}$$

α_c is useful only for the user who may use it to take into account combined bba's reliability degree in decision process.

Equations (3.7) and (3.8) are different: the first one computes source's reliability which is the mean of all its relative reliabilities and the second one computes combination's reliability which is the mean of relative reliabilities of combined bbas.

Example 3.8 Suppose that we have three evidential databases to integrate, each evidential database belongs to a source (namely s_1, s_2 and s_3). These evidential databases

Table 3.13: Global reliabilities

Source	**Conflicts**	**Relative reliabilities**	**Global reliability**
s_1	0.5	0.0858	0.1134
	0.39	0.141	
s_2	0.5	0.0858	0.1119
	0.395	0.1380	
s_3	1	0	0.0315
	0.561	0.063	

are used to estimate conflict values and relative reliabilities of each source. Let us take the second bbas of tables 3.10, 3.11, 3.12:

Table 3.14: Combination's reliability

Source	**bba**	**Relative reliability**	**Combined bba (Dempster's rule of combination)**	**Combination's reliability**
s_1	$P(0.5)$ $H(0.5)$	0.141	$P(0.33)$ $H(0.67)$	0.114
s_2	$P(0.2)$ $H(0.6)$ $\{P \cup H\}(0.2)$	0.1380		
s_3	$M(0.6)$ $\{P \cup H\}(0.4)$	0.063		

■

3.3.5 Discounting

Once global reliability is estimated, this measure is used to discount all bbas supplied by the source having this reliability value, we suggest to use methods of mass discounting as well as plausibility discounting described in Chapter 2.

Thus, we begin by calling back both methods of discounting.

Mass discounting

Source's global reliability (α for short) is used to discount all belief functions (bbas) provided by this source. The discounting operator used in this method is given by:

$$\begin{cases} m^{\Omega,\alpha}(A) = \alpha \times m^{\Omega}(A) & \text{for } A \subset \Omega \\ m^{\Omega,\alpha}(\Omega) = (1-\alpha) + (\alpha \times m^{\Omega}(\Omega)) \end{cases} \tag{3.9}$$

Example 3.9 Let us take the same tables described in example 3.7 and discount stored bbas using global reliabilities. Reliabilities of s_1, s_2 and s_3 are respectively equal to 0.1134, 0.1119 and 0.0315. Discounted evidential databases are given in tables 3.15, 3.16 and 3.17.

Table 3.15: Discounted EDB of s_1 using mass discounting operator

id	Time	Target
1	t_1	$P(0.1134)$ $\Omega(0.8866)$
2	t_2	$P(0.0567)$ $H(0.0567)$ $\Omega(0.8866)$

Table 3.16: Discounted EDB of s_2 using mass discounting operator

id	Time	Target
1	t_1	$P(0.1119)$ $\Omega(0.8881)$
2	t_2	$P(0.02238)$ $H(0.06714)$ $\{P \cup H\}(0.02238)$ $\Omega(0.8881)$

■

Plausibility discounting

The operator described above weakens or strengthens bbas, mass by mass, proportionally to source's reliability. Therefore, this operator does not affect focal elements but does change only masses. Thus, we suggest to use plausibility discounting rather than

Table 3.17: Discounted EDB of s_3 using mass discounting operator

id	Time	Target
1	t_1	$P(0.0315)$ $\Omega(0.9685)$
2	t_2	$M(0.0189)$ $H(0.0126)$ $\Omega(0.9685)$

mass discounting.

Plausibility discounting proposed in (Zeng and Wu, 2007) is in three steps; in the first step plausibility function is computed from bbas:

$$\begin{aligned} pl : 2^{\Omega} &\rightarrow [0,1] \\ A &\mapsto \sum_{A \cap B \neq \emptyset} m^{\Omega}(B) \end{aligned} \tag{3.10}$$

In the second step, plausibility functions are discounted:

$$pl^{\alpha}(A) = [pl(A)]^{\alpha} \quad \forall A \subseteq \Omega \text{ and } A \neq \emptyset \tag{3.11}$$

and finally, discounted bbas are computed from discounted plausibility functions:

$$\begin{cases} m^{\Omega,\alpha}(B) = \sum_{B \subseteq A} (-1)^{|A|-|B|+1} pl^{\alpha}(\bar{B}) \\ m^{\Omega,\alpha}(\emptyset) = 1 - pl^{\alpha}(\Omega) \end{cases} \tag{3.12}$$

Example 3.10 For the example 3.7, discounted bbas using plausibility discounting are given in tables 3.18, 3.19 and 3.20.

Table 3.18: Discounted EDB of s_1 using plausibility discounting operator

id	Time	Target
1	t_1	P
2	t_2	$P(0.0756)$ $H(0.0756)$ $\{P \cup H\}(0.8488)$

■

Table 3.19: Discounted EDB of s_2 using plausibility discounting operator

id	Time	Target
1	t_1	P
2	t_2	$P(0.0247)$ $H(0.0975)$ $\{P \cup H\}(0.8779)$

Table 3.20: Discounted EDB of s_3 using plausibility discounting operator

id	Time	Target
1	t_1	M
2	t_2	$\{P \cup H\}(0.016)$ $M(0.0285)$ $\Omega(0.9556)$

For simplicity of calculation, we have computed relative conflict values (conflict computed from only one bba) for each source then we used the mean to estimate source's global reliability. This method reduces the number of uses of equation (3.2), thus it is used only once rather than many times.

3.4 Conclusion

In this chapter, we have proposed to estimate the conflict degree of a source on the bases of all its bbas. This conflict degree is evaluated for each source against all the others according to each bba supplied by this source. These conflict degrees are used to compute the source's relative reliabilities.

The mean of all source's relative reliabilities is considered to be the source's global reliability. This global reliability value is used either to discount bbas or to discount plausibility functions. These methods will be tested in Chapter 4 on real radar data.

4
Experiments

4.1 Introduction

In the previous chapter, we presented our method used for source's reliability estimation. This method computes the conflict of a bba supplied by a source against all the other bbas (provided by different sources), therefore we estimate the conflict of a source against all the other sources.

Most of the time, a source provides a great number of bbas, thus we suggest to estimate the conflict of a source according to each bba and to use the mean of this conflict values as the source's global conflict. This global conflict value is used to compute the global reliability of each source.

We have implemented this method using Matlab tools and real radar data collected in the anechoic chamber of the ENSIETA (Brest, France). The main purpose of this method is to improve source's reliability, thus global reliabilities are estimated before and after discounting. We aim also to insure the same level of relative reliabilities after discounting; therefore all bbas provided by a source have the same level of relative reliability which is source's global reliability.

In this chapter, the second section is dedicated to describe data used in our experiments. In the third section, we estimate the global reliability of each source on the bases of all its collected bbas. These reliability values will be used to discount all bbas in the fourth section and also to discount plausibility functions in the fifth section. Both methods of mass discounting and plausibility discounting will be compared in the last section of this chapter.

4.2 Data description

To test our method described in the previous chapter, we considered a database containing radar data. The real data were obtained in the anechoic chamber of ENSIETA (Brest, France) using a target radar sensor with different angular positions.
The acquisition process is described in (Martin and Radoi, 2004) and a model of database used for storing corresponding frequency data is proposed in (Toumi, 2007).

Each database contains 250 frequencies obtained on angular position about 60° and using a frequency band of 6 GHZ.

We considered five radar targets (namely Mirage, F14, Rafale, Tornado, Harrier) and three classifiers considered as sources. These classifiers which are: fuzzy K-nearest neighbour, belief K-nearest neighbour (Denœux, 1995) and neural network are used to analyze and classify frequencies data in order to produce 250 bbas. These 250 bbas (for each source) are stored in three tables which we use to test our method.

Figure 4.1 describes data acquisition process. A model of a target and a sensor were placed in the anechoic chamber. The sensor gives a frequency when detecting the target from different angular position.

Our purpose is to integrate the three tables by combining these 250 bbas of each source in order to have only one table. Combining three tables in only one will help users when decision making.

We also aim to ensure the same level of relative reliability for all bbas provided by the same source and this relative reliability level is the source's one.
When all source's bbas have the same level, cases when source is wrong are discounted and user may use all bbas without carrying about mistakes because they are corrected. To be sure that all bbas provided by the same source have the same level, we have to reduce the variance of relative reliabilities.

Also, enriching databases by adding extra information about sources' initial reliability degrees (sources' reliability degrees before discounting) to be used for maintaining databases if new data are added and have to be integrated. Adding combinations' degree of reliability will inform users about the pertinence of combined bbas especially that the user will use the integrated database rather than the initial ones separately.

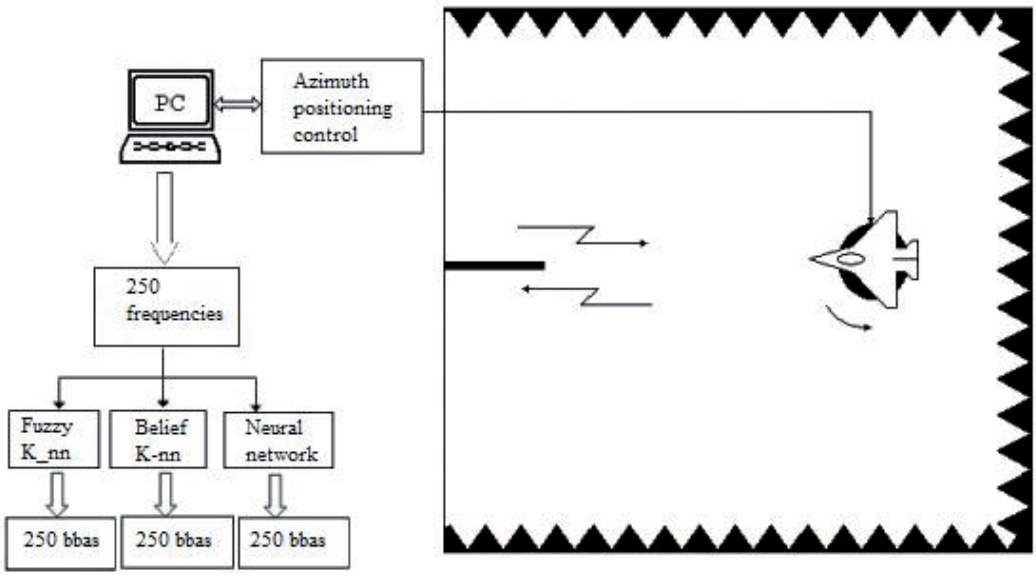

Figure 4.1: Data acquisition

4.3 Global reliability estimation

Our method consists on estimating the global reliability of a source from all its bbas. We have three tables containing each one 250 bbas, each bba is a classification result of one classifier (considered as a source).

From the three tables containing 250 bbas each one, a conflict measure is attributed to every bba using distance type 1 and distance type 2 with combination rules described in Chapter 1. Conflict estimation method is described in the previous chapter. These conflict values are used to estimate the relative reliability of each bba as described in the previous chapter.
Therefore, we obtained 250 relative reliability values for each source (fuzzy K-nearest neighbour, belief K-nearest neighbour and neural network).

Table 4.1 contains the minimum, maximum and mean of relative reliabilities for each source with distance type 1 for conflict estimation and $\lambda = 1/2$.

Discounting with the minimal reliabilities which are very small (0.073, 0.029 and 0.09) will reduce bbas to the case of total ignorance, discounting with the maximal reliabilities which are high (0.741, 0.676 and 0.719) does not really affect bbas, and

Table 4.1: Minimum, maximum and mean of relative reliabilities

Source	**Max**	**Min**	**Mean**
Fuzzy K-nearest neighbour	0.741	0.073	0.313
Belief K-nearest neighbour	0.676	0.029	0.28
Neural network	0.719	0.09	0.205

finally discounting with the mean of relative reliabilities reduces the conflict and also keeps the structure of bbas unchanged.

Therefore, we choose the mean of these 250 relative reliabilities as source's global reliability.

For simplicity of calculation, we have computed 250 conflict values for each source then we used the mean to estimate source's reliability. This method reduces the number of use of the equation (3.2) described in the previous chapter, thus it is used only once rather than 250 times.

Table 4.2 contains initial conflicts computed using distance type 1 and distance type 2 (using conjunctive rule of combination, Dempster's rule of combination, Murphy's rule of combination (Mean), Dubois and Prade's rule of combination and Yager's rule of combination) for each source. The initial conflict of a source is the mean of 250 relative conflict values (the relative conflict value is computed on the bases of only one bba).

This global conflict value is used to estimate source's global reliability using the following equation:

$$\alpha_j = (1 - Conf(s_j, s)^{\lambda})^{\frac{1}{\lambda}} \tag{4.1}$$

λ is a not null real.
Equation (4.1) is detailed in the previous chapter.

Reliability estimation depends on the value of conflict and also on λ, thus figures 4.2, 4.3 and 4.4 present initial global reliabilities for the three sources (fuzzy K-nearest neighbor, belief K-nearest neighbor and neural network) for different values of λ.

Figures 4.2, 4.3 and 4.4, show that reliability increases when λ is greater for the same level of conflict. In these three figures, initial conflict is the same for each plot

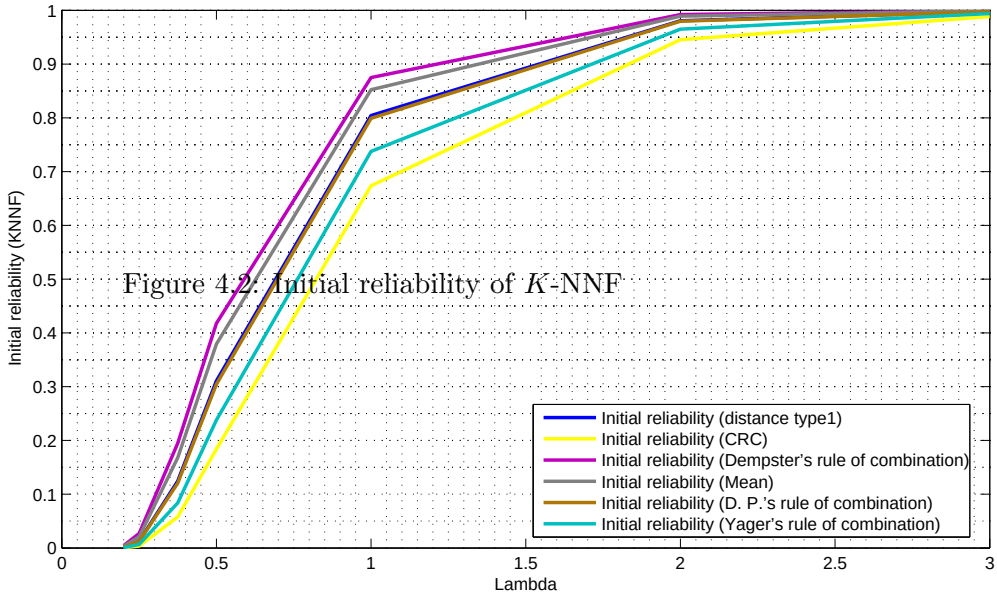

Figure 4.2: Initial reliability of K-NNF

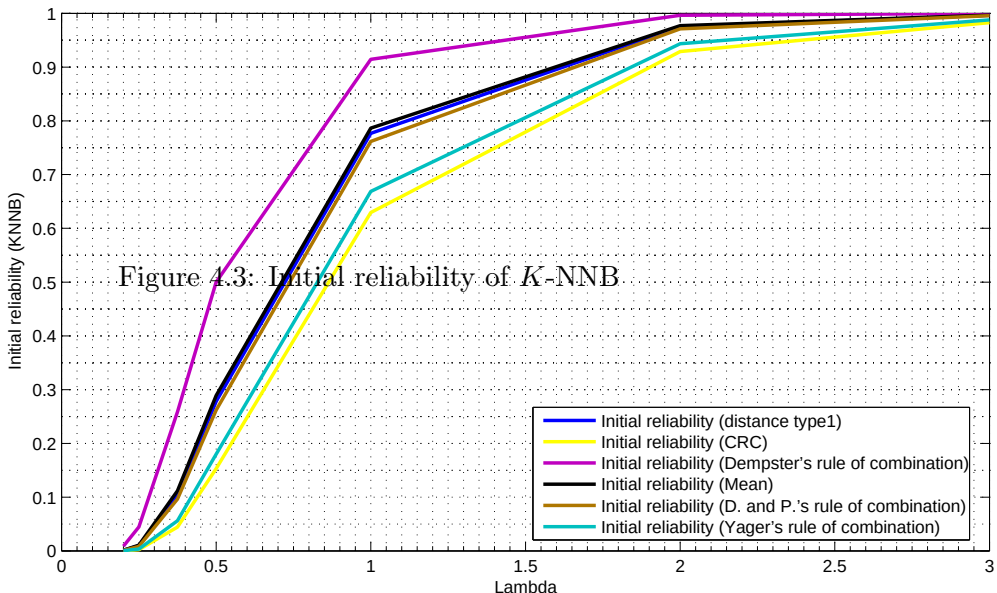

Figure 4.3: Initial reliability of K-NNB

Table 4.2: Initial Conflict

Source	**Type of distance**	**Combination rule**	**Conflict**
Fuzzy K-nearest neighbour	Type 1	-	0.196
	Type 2	CRC	0.326
		Dempster's rule of combination	0.125
		Mean	0.148
		D. and P.'s rule of combination	0.201
		Yager's rule of combination	0.262
Belief K-nearest neighbour	Type 1	-	0.223
	Type 2	CRC	0.371
		Dempster's rule of combination	0.086
		Mean	0.214
		D. and P.'s rule of combination	0.238
		Yager's rule of combination	0.331
Neural network	Type 1	-	0.301
	Type 2	CRC	0.331
		Dempster's rule of combination	0.334
		Mean	0.292
		D. and P.'s rule of combination	0.319
		Yager's rule of combination	0.316

but the reliability is increasing when λ increases and its limit is 1.

4.4 Mass discounting

Once the source's global reliability is estimated, all bbas provided by this source can be discounted using the classical mass discounting operator described in details in Chapter 2 and defined as follows:

$$\begin{cases} m^{\Omega,\alpha}(A) = \alpha \times m^{\Omega}(A) & \text{for } A \subset \Omega \\ m^{\Omega,\alpha}(\Omega) = (1-\alpha) + (\alpha \times m^{\Omega}(\Omega)) \end{cases} \tag{4.2}$$

After discounting all bbas using their sources' reliabilities, we re-estimated conflicts and global reliabilities of these sources. In figures 4.5, 4.6 and 4.7, reliability improvement rates are presented in the purpose of verifying that this method improves reliabilities after discounting.

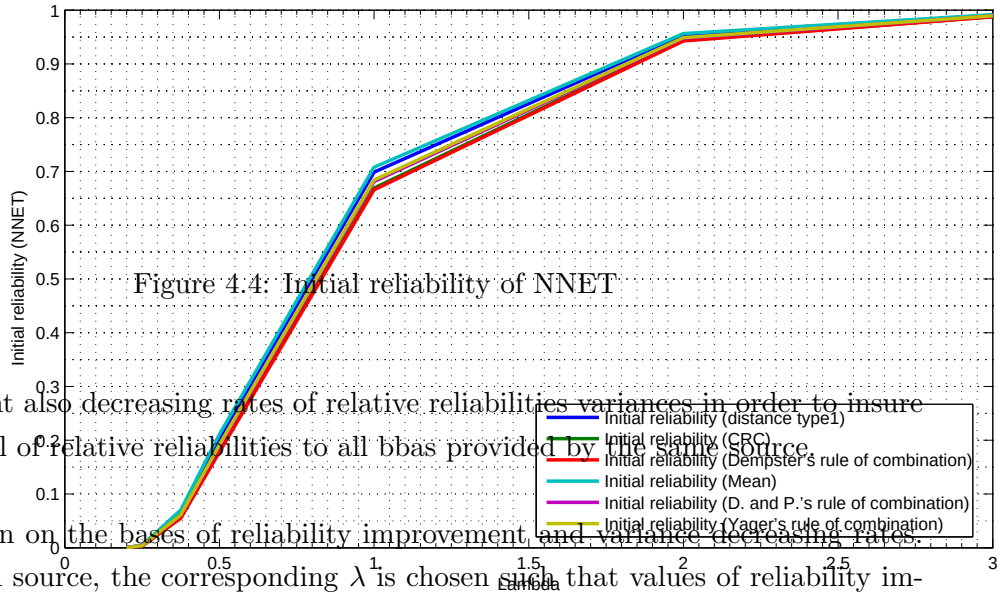

Figure 4.4: Initial reliability of NNET

We present also decreasing rates of relative reliabilities variances in order to insure the same level of relative reliabilities to all bbas provided by the same source.

λ is chosen on the bases of reliability improvement and variance decreasing rates. Thus for each source, the corresponding λ is chosen such that values of reliability improvement and variance decreasing rates are positive meaning that global reliability is improved and also relative reliabilities have the same level.

Figures 4.5, 4.6 and 4.7 show that for $\lambda \leq 0.375$, reliability improvement and variance decreasing rates are positive. Therefore, we suggest to use $\lambda = 0.375$ for reliability estimation from conflict value because the conflict increases with the growth of λ. Thus, λ has to be as great as possible to discount bbas at least.

Table **??** resumes reliability values before and after discounting as well as variance of relative reliabilities decreasing rates for each source using both types of distance and using $\lambda = 0.375$ for reliability estimation.

For $\lambda = 0.375$, reliabilities are improved whatever the type of distance and the combination rule. Relative reliabilities' decreasing rates are almost equal to 100%,

Table 4.3: Reliabilities before and after discounting using mass discounting operator

Source	**Type of distance**	**Combination rule**	**Initial reliability**	**Reliability after discounting**	**Variance decreasing rate**
Fuzzy K-nearest neighbour	Type 1	-	0.124	0.397	100%
	Type 2	CRC	0.058	0.422	100%
		Dempster's rule	0.195	0.234	98.5%
		Mean	0.168	0.303	99.1%
		D. and P.'s rule	0.121	0.433	99.4%
		Yager's rule	0.084	0.456	100%
Belief K-nearest neighbour	Type 1	-	0.105	0.412	99.4%
	Type 2	CRC	0.044	0.361	100%
		Dempster's rule	0.258	0.343	96.7%
		Mean	0.111	0.529	98.8%
		D. and P.'s rule	0.096	0.337	100%
		Yager's rule	0.056	0.323	100%
Neural network	Type 1	-	0.067	0.333	99.7%
	Type 2	CRC	0.056	0.357	99.7%
		Dempster's rule	0.055	0.065	99%
		Mean	0.07	0.288	99.7%
		D. and P.'s rule	0.06	0.184	99.7%
		Yager's rule	0.061	0.285	99.7%

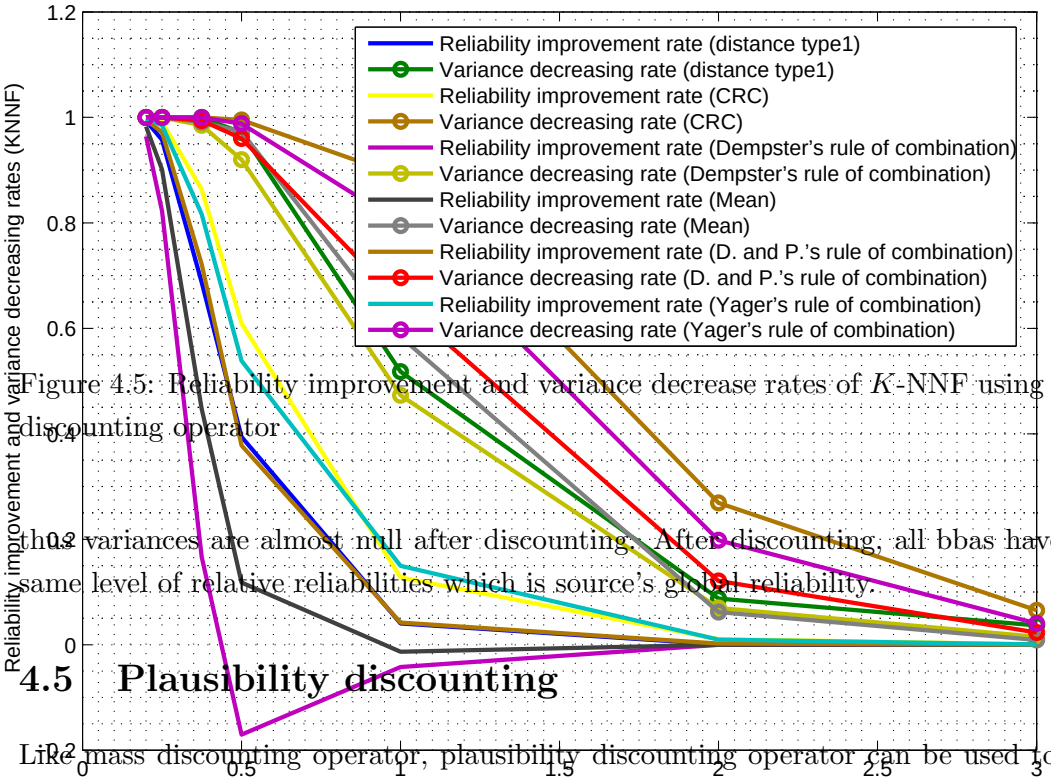

Figure 4.5: Reliability improvement and variance decrease rates of K-NNF using mass discounting operator

thus variances are almost null after discounting. After discounting, all bbas have the same level of relative reliabilities which is source's global reliability.

4.5 Plausibility discounting

Like mass discounting operator, plausibility discounting operator can be used to discount all plausibility functions computed from all bbas. This operator was also detailed in Chapter 2. To call back this operator, it is on three steps:

- **Step 1:** Plausibility functions are computed from bbas:

$$\begin{aligned} pl : 2^{\Omega} &\to [0,1] \\ A &\mapsto \sum_{A \cap B \neq \emptyset} m^{\Omega}(B) \end{aligned} \tag{4.3}$$

- **Step 2:** Discounting plausibility functions:

$$pl^{\alpha}(A) = [pl(A)]^{\alpha} \quad \forall A \subseteq \Omega \text{ and } A \neq \emptyset \tag{4.4}$$

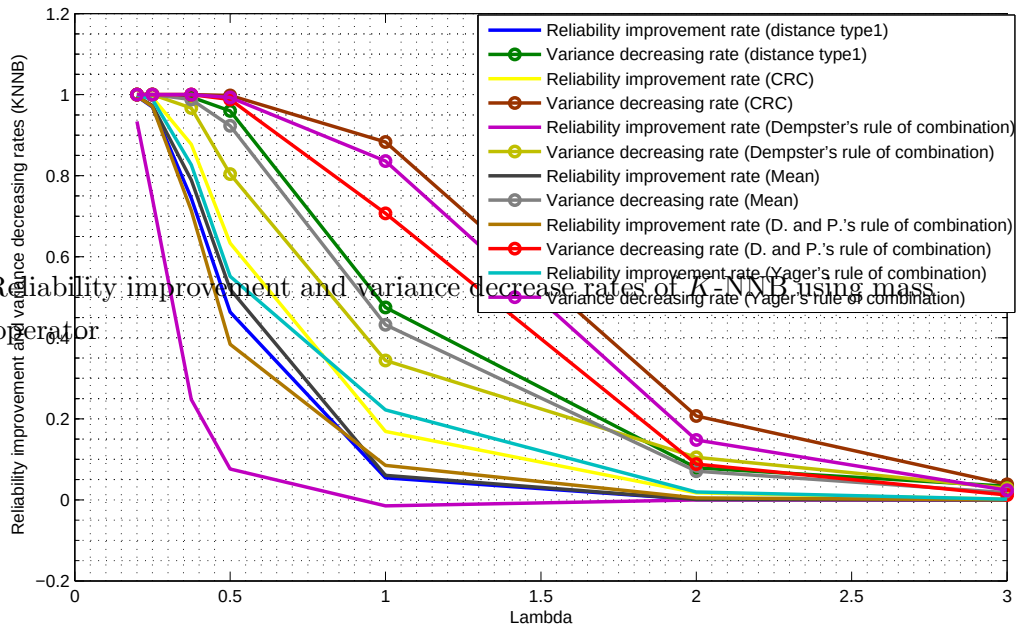

Figure 4.6: Reliability improvement and variance decrease rates of K-NNB using mass discounting operator

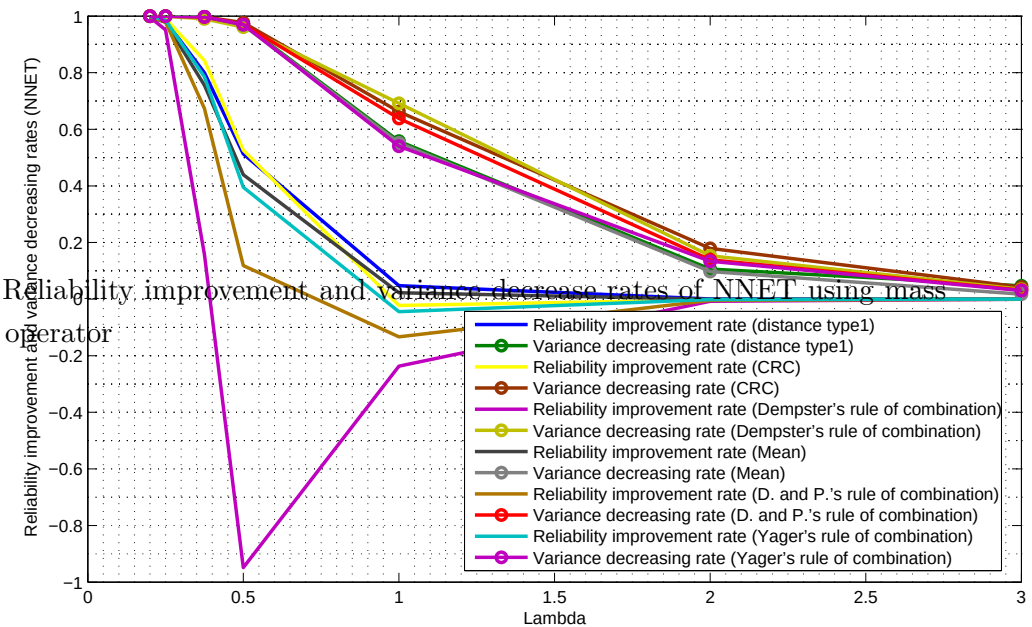

Figure 4.7: Reliability improvement and variance decrease rates of NNET using mass discounting operator

- **Step 2:** Discounted bbas are converted back from discounted plausibility functions:

$$\begin{cases} m^{\Omega,\alpha}(B) = \sum_{B \subseteq A} (-1)^{|A|-|B|+1} pl^{\alpha}(\bar{B}) \\ m^{\Omega,\alpha}(\emptyset) = 1 - pl^{\alpha}(\Omega) \end{cases} \tag{4.5}$$

After discounting all bbas using their sources' reliabilities, we re-estimated conflicts and global reliabilities of these sources. In figures 4.8, 4.9 and 4.10, reliability improvement and relative reliabilities' variances decrease rates are presented in the purpose of verifying if this operator improves reliabilities and also to verify if relative reliabilities are the same after discounting.

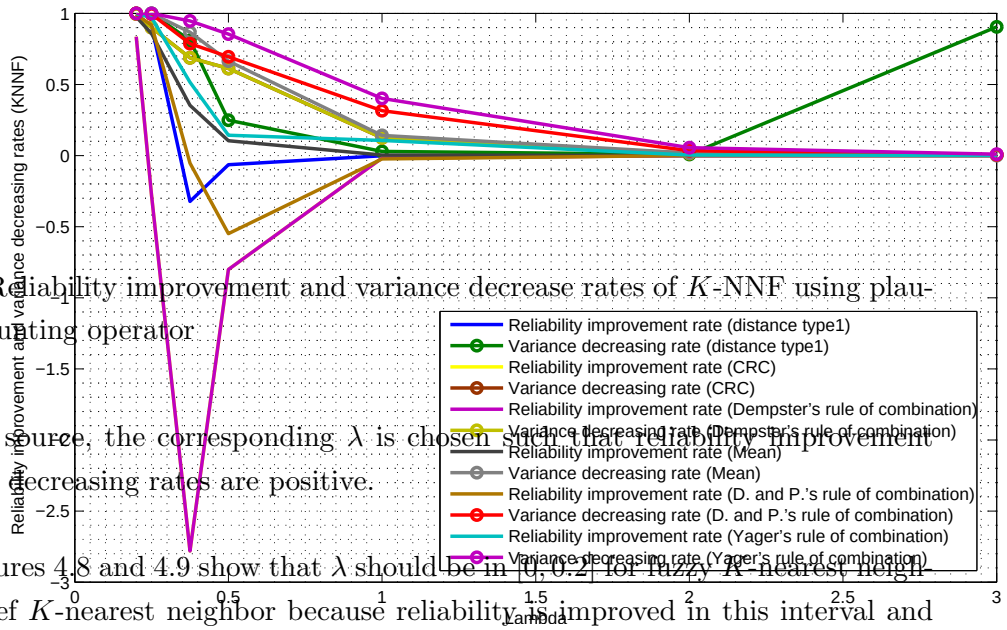

Figure 4.8: Reliability improvement and variance decrease rates of K-NNF using plausibility discounting operator

For each source, the corresponding λ is chosen such that reliability improvement and variance decreasing rates are positive.

Thus, figures 4.8 and 4.9 show that λ should be in $[0, 0.2]$ for fuzzy K-nearest neighbor and belief K-nearest neighbor because reliability is improved in this interval and also variance of relative reliabilities are very small (almost 0). λ must be chosen as great as possible, thus $\lambda = 0.2$ is chosen for these sources.

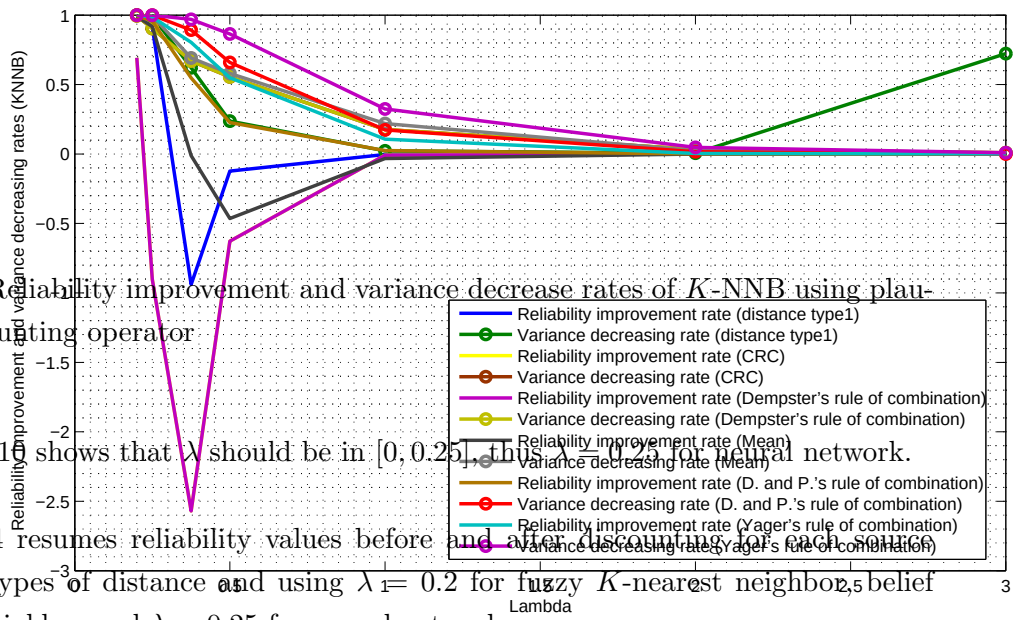

Figure 4.9: Reliability improvement and variance decrease rates of K-NNB using plausibility discounting operator

Figure 4.10 shows that λ should be in $[0, 0.25]$, thus $\lambda = 0.25$ for neural network.

Table 4.4 resumes reliability values before and after discounting for each source using both types of distance and using $\lambda = 0.2$ for fuzzy K-nearest neighbor, belief K-nearest neighbor and $\lambda = 0.25$ for neural network.

For these values of λ, reliabilities are improved and relative reliabilities' variances are almost null after discounting whatever the type of distance and the combination rule. After discounting, all bbas have the same level of relative reliabilities which is source's global reliability.

4.6 Comparison of mass and plausibility discounting

To compare both methods of mass and plausibility discounting, reliability improvement rates, presented in table 4.5, have to be compared rather than reliabilities after discounting because initial reliabilities are different (because values of λ are different).

For both methods relative reliabilities' variances are very small (almost 0), thus

Table 4.4: Reliabilities before and after discounting using plausibility discounting operator

Source	**Type of distance**	**Combination rule**	**Initial reliability**	**Reliability after discounting**
Fuzzy K-nearest neighbour	Type 1	-	0.002	0.156
	Type 2	CRC	0.005	0.028
		Dempster's rule	0.005	0.028
		Mean	0.003	0.129
		D. and P.'s rule	0.0016	0.175
		Yager's rule	0.0007	0.264
Belief K-nearest neighbour	Type 1	-	0.001	0.135
	Type 2	CRC	0.009	0.029
		Dempster's rule	0.009	0.029
		Mean	0.0013	0.156
		D. and P.'s rule	0.001	0.204
		Yager's rule	0.0003	0.24
Neural network	Type 1	-	0.005	0.11
	Type 2	CRC	0.0033	0.0076
		Dempster's rule	0.0033	0.0076
		Mean	0.005	0.09
		D. and P.'s rule	0.0038	0.062
		Yager's rule	0.0039	0.126

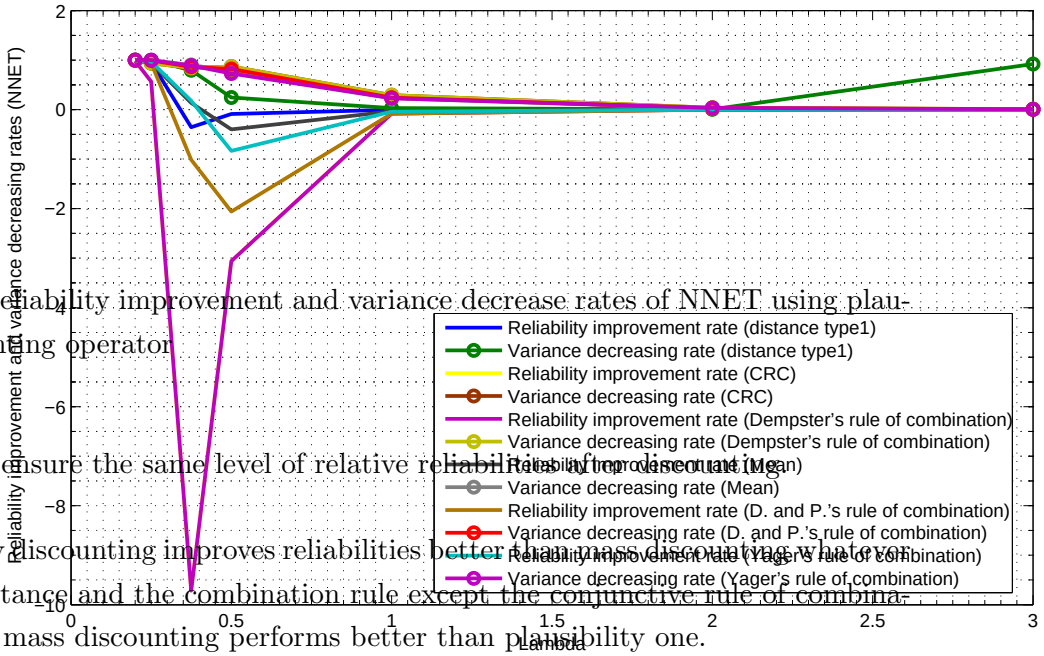

Figure 4.10: Reliability improvement and variance decrease rates of NNET using plausibility discounting operator

both methods ensure the same level of relative reliabilities after discounting.

Plausibility discounting improves reliabilities better than mass discounting whatever the type of distance and the combination rule except the conjunctive rule of combination for which mass discounting performs better than plausibility one.

Mass discounting operator discounts bbas less than plausibility one because of the choice of λ which is greater for mass discounting therefore, bbas are less discounted but plausibility discounting operator does not affect all the discount factor to the total ignorance.

4.7 Conclusion

In this chapter, we have presented our method of sources' reliability estimation on the bases of all its bbas which are stored in an evidential database. These reliability values are used to discount bbas as well as plausibility functions. Both methods of mass and plausibility discounting gave good results and improved sources' reliabilities after

Table 4.5: Reliability improvement percentages for mass and plausibility discounting operator

Source	**Type of distance**	**Combination rule**	**Reliability improvement (plausibility discounting)**	**Reliability improvement (mass discounting)**
Fuzzy K-nearest neighbour	Type 1	-	98.92%	68.7%
	Type 2	CRC	83.61%	86.34%
		Dempster's rule	83.61%	16.63%
		Mean	97.48%	44.69%
		D. and P.'s rule	99.11%	72.15%
		Yager's rule	99.73%	81.63%
Belief K-nearest neighbour	Type 1	-	99.14%	74.45%
	Type 2	CRC	69.25%	87.73%
		Dempster's rule	69.25%	24.75%
		Mean	99.15%	78.93%
		D. and P.'s rule	99.53%	71.39%
		Yager's rule	99.87%	82.68%
Neural network	Type 1	-	95.91%	79.97%
	Type 2	CRC	56.61%	84.28%
		Dempster's rule	56.61%	15.78%
		Mean	94.38%	75.57%
		D. and P.'s rule	93.81%	67.36%
		Yager's rule	96.89%	78.52%

discounting.

The purpose of sources' reliability estimation is to discount related evidential information before combining them. Combining discounted evidential information is used to integrate different evidential databases in the purpose of reducing the quantity of data to be stored and also helping the users when making decisions.

Conclusion

Managing uncertainty is a vast domain of research, many theories are used in this context such as the theory of belief functions. This theory is a strong tool used for representing and managing uncertainty and also to combine several uncertain information in order to limit uncertainty or to reduce it. The main problem appearing when combining several evidential information is the conflict which may appear reflecting the contrast between their sources. This conflict can be managed in the combination by using adequate combination rules and can also be eliminated or reduced before combination by taking into account the sources' reliability degrees. Discounting evidential information before combining reduces the conflict which may appear after combining but this operator needs an *a prior* knowledge of source's reliability degree.

Many researches are focused on estimating or learning source's reliability degree, our work is situated in this context because we have presented a method of source's reliability estimation which aims to estimate the degree of reliability of a source from all its evidential information which are stored in an evidential database. This method estimates the conflict of a source with all other available sources and uses it to compute the reliability of that source.
Source's reliability estimation using its evidential database is done in the purpose of discounting all evidential information stored in this evidential database before integrating them with other ones.

In future work, we will tend to investigate different horizons in order to improve this work:

- We will study in details distance measures in order to propose a distance on plausibility functions in the purpose of using it to estimate source's reliability when using plausibility discounting operator. Plausibility functions have some

common properties with belief functions because both functions are a mapping from 2^{Ω} to $[0, 1]$ but the sum of plausibilities affected to focal elements are not equal to 1.

- We will attack the dynamic part of combination because a sensor detects an approaching target and provides different evidential information concerning the same object but detected from different distances. Dynamic combination reduces also the quantity of data to be stored and also improve sensor's detecting results.

- Finally, source's reliability can be estimated according to each hypothesis from the frame of discernment thus a sensor can be more reliable when detecting some targets and less reliable for others. Source's reliability can be dependent on the target itself and also on the weight of that target for him.

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