Université de Tunis

Institut Supérieur de Gestion

MASTER RECHERCHE

Spécialité

Sciences et Techniques de l'Informatique Décisionnelle (STID)

Option

Informatique et Gestion de la Connaissance (IGC)

# PREFERENCE REPRESENTATION IN THE POSSIBILISTIC FRAMEWORK

Héla Gouider

| | | |
|---|---|---|
| Nahla Ben Amor | Professeur, ISG Tunis | Directeur du mémoire |
| Henri Prade | Directeur de recherche, IRIT Toulouse | Co-directeur |

Laboratoire/Unité recherche: LARODEC

2013-2014

# Acknowledgment

I would like to express my sincere gratitude to my supervisor Nahla Ben Amor who provided me a comfortable way to launch into research. I esteem her for her continuous guidance and encouragement to achieve excellence. Her competence and especially her patience have allowed me to improve.

I would like to express my gratitude to my co-supervisor Henri Prade. He inspired my scientific curiosity through many worthwhile and helpful discussions. His permanent guidance led to a wonderful atmosphere resulting in the work at hand. I think that I am really fortunate having met him.

My gratitude goes also to Didier Dubois for his interest to my work and his constructive and valuable discussions. He helped me in many steps for the fulfillment of my research.

A very special thanks to my father who supported me during all these years. For his advice and encouragements. He is my inspiration.

Of course, many thanks to my mother who was always there for me, helping me each time to move forward.

Last but not least, a special wink to my two sisters for what they have done for me during this research.

My love to all my friends. I wish you all the best.

Many thanks go to members of my laboratory LARODEC.

# Contents

# List of Figures

# List of Tables

# Introduction

Preferences are usually expressed by means of pieces of information in a local manner, rather than as a complete preorder between the different possible states of the world. This state of fact has led AI researchers to propose compact representation formats for preferences and procedures for computing a plausible ranking between completely described situations from such representations, in the last fifteen years. Conditional preference networks (CP-nets for short) (Boutilier, Brafman, Domshlak, Hoos, & Poole, 2004) have emerged as a popular reference setting for representing preferences, leading to different refinements (Brafman & Domshlak, 2002; Wilson, 2011), as well as some alternative approaches (Bienvenu, Lang, & Wilson, 2010; Dubois, Kaci, & Prade, 2006; Kaci & van der Torre, 2008; Benferhat, Dubois, & Prade, 1999). See (Domshlak, Hüllermeier, Kaci, & Prade, 2011) for a brief overview. Inspired from Bayesian nets, CP-nets inherit from their graphical nature, and besides, rely on a simple, apparently natural principle, named *ceteris paribus*, which allows to extend any contextual preference "in context $c$, I prefer $a$ to $\neg a$" (denoted for short $c : a \succ \neg a$), to any particular specification $b$ of the other variables used for describing the considered situations, i.e., the preference is understood as $\forall b, cab$ is preferred to $c\neg ab$.

The CP-net approach perfectly exemplifies the ingredients needed for a satisfactory completion of preferences, stated in a possibly conditional manner, into a preorder useful for a user: i) a simple representation setting, preferably having a graphical counterpart for elicitation ease, ii) a natural principle for making explicit the preferences between completely described situations, and iii) an algorithm for determining how to compare two complete situations according to the existence of a path of worsening flips linking them.

In spite of their appealing features, CP-nets have some limitations. First, there exist preorders that make sense and for which there does not exist any CP-net that can be associated to them. Moreover, they tend to force some debatable priorities between preferences associated to nodes in the CP-nets, beyond what is really expressed by the preferences one starts with (Dubois, Prade, & Touazi, 2013b; Kaci & Prade, 2008). Therefore, we assumed that having a synthetic comparison between all the existing methods is convenient. First, to underline the different characteristics of each method. Second, to highlight their limitations. Then to propose an approach that is able to overcome almost all the problems.

In this report, we advocate possibilistic networks as a valuable tool for representing preferences. First, possibilistic nets are an exact counterpart of Bayesian nets in possibility theory, based on a possibilistic Bayesian-like conditioning rule. Although they have been only used for uncertainty modeling until now, they can serve preference modeling purposes as well, as shown in the following, without having the CP-nets limitations detected. we also provide a comparison between different ordering models of preferences.

This master thesis is organized into 4 chapters. Chapter 1 provides a background on CP-nets and their algorithms. Then chapter 2 proposes a general overview of the possibility theory and the different possibilistic formats for preference modeling. Chapter 3 presents a comparative discussion between the possibilistic framework for preferences and the CP-nets where the main differences are outlined. Finally, chapter 4 describes the proposed model and establishes some properties.

Chapter 1

# CP-nets: Reasoning under the Ceteris Paribus principle

## 1.1 Introduction

In several domains it is very important to assess the users'preferences. Such representation of preference ordering form an important task of the automated decision tools. Generally, all the available decisions or actions are fixed and the only inconstant component in the process is the preferences on whose behalf a decision is being made. This task is mainly arduous and many sophisticated techniques exist to achieve it. Ideally, a preference representation should capture statements that are natural to the users to asses and reasonably compact.

One of the most popular models for preferences is the Conditional Preference network (CP-net for short) (Boutilier et al., 2004) which provides a compact way to express preferences using a graphical representation. CP-nets rely on the Ceteris Paribus principle which means that every thing else is equal. For instance, if a person says '*I prefer to have a cat if I live in a flat*' it asserts that given two similar flats one with a cat and the other with a dog, the house with a cat will be preferable. This tells us nothing about his preferences if he is living in a villa. Such representation is natural, compact and intuitive.

This chapter is organized as follows: Section 1.2 and 1.3 introduce some definitions on preference independence used in this chapter. Section 1.4 is devoted to the the logical and graphical representation of the CP-net. Section 1.5 details the different queries executed on the CP-net. Finally, section 1.6 presents some extensions of the CP-net more precisely a probabilistic version so-called the Probabilistic Conditional Preferential network (PCP-nets for short).

## 1.2 Notations and definitions

We first give some notations and definitions that are going to be used for the rest of the report:

- $V = \{A_1, A_2, \dots, A_i\}$ a set of variables;
- $D_A = a_1, \dots, a_n$ denotes the finite domain of the variable $A$;
- $a_i$ denotes any instance of $A_i$;
- $X, Y, Z, \dots$ denote subsets of variable form $V$;
- $D_X = \times_{A_i \in X} D_{A_i}$ denotes the cartesian product of domains of variables in $X$ and called assignment;
- $x$ denotes any instance of $X$, if $X = \{A_1, \dots, A_n\}$ then $x = (a_i, \dots, a_n)$;
- $\Omega = \times_{A_i \in V} D_{A_i}$ denotes the universe of discourse, which is the cartesian product of all variable domains in $V$;
- Each element $\omega \in \Omega$ is called interpretation, a possible world or a state of $\Omega$;
- $p, q, \dots$ denote subclasses of $\Omega$ (called events) and $\neg p$ denotes the complementary set of $p$ i.e. $\neg p = \Omega - p$;
- $p \wedge q$ (respectively $p \vee q$) denotes the intersection (union) of $p$ and $q$.

## 1.3 Preferences and ordering

Since we are talking about preferences and orderings, let us first introduce the notion of order. Note that the aim of preference models is to generate an order between all the possible interpretations. Let $\Omega$ be a set of alternative choices. Note that the order induced by the CP-nets is a partial preorder.

**Definition 1.1.** ***Ordering:*** *Let $\Omega$ and $\preceq$ be respectively a universe of discourse and a binary relation on $\Omega$. $\preceq$ is considered as a partial order if it satisfies three properties:*
*$\forall$ a, b a and c $\in \Omega$:*

- *Reflexivity: $a \succeq b$ or $b \preceq a$;*
- *Asymmetry: if $a \succeq b$ and $b \preceq a$ then $a = b$;*
- *Transitivity: if $a \succeq b$ and $b \succeq c$ then $a \succeq c$.*

**Definition 1.2.** *(Boutilier et al., 2004) A preference ranking (or preference ordering) is a total preorder ($\preceq$) over a set of values: s.t. $\omega_1 \preceq \omega_2$ ( $\omega_1$, $\omega_2 \in \Omega$) means that the interpretation $\omega_1$ is equally or more preferred than $\omega_2$.*

We will use the term preference ordering in the remaining of this report.

When asserting the preferences the user may express his preferences towards some variable in a conditional manner. It means that some variables are dependent of other variables values.

**Definition 1.3.** *A set of variables $X \subseteq V$ is preferentially independent from its complement $Y = \Omega/X$ iff, $\forall x_1$, $x_2 \in Asst(X)$ and $\forall$ $y_1$, $y_2 \in Asst(Y)$, we have:*

$$x_1y_1 \preceq x_2y_1 \text{ iff } x_1y_2 \preceq x_2y_2. \tag{1.1}$$

This relation corresponds to a Ceteris Paribus relation. It means that 'I prefer $x_1$ to $x_2$ with every thing being equal. In the same way we can define the conditional preferential dependence. Assume that we have 3 sets of variables $X$, $Z$ and $Y$. If $Z \in V$ is conditionally preferentially dependent on $X$, and $X$ on $Y$, then $Y$ is preferentially independent of $Z$.

These kinds of preferences are relatively weak because we can not apply special trade-offs. In fact, consider two variables $A$ and $B$ preferentially independent, note that we can not define the relative importance between them. A solution of this problem was proposed in (Brafman & Domshlak, 2002).

## 1.4 CP-net representation

CP-nets (Boutilier et al., 2004; Boutilier, Brafman, Hoos, & Poole, 1999) are graphical and powerful models for representing and analyzing preferences. They are characterized by their clarity and their efficient management and storage of the information. CP-nets express the conditional preferential relations by means of graphical dependence represented by the edges. They capture only qualitative statements.

**Definition 1.4.** *A CP-net over variables* $V = \{A_1, A_2, \ldots, A_n\}$ *with domains* $D(A_1), \ldots, D(A_n)$ *is a directed graph where:*

- *Each variable* $A$*, associated with a domain* $D(A) = \{a_1, a_2, ..., a_n\}$ *of values it can take, is represented by a node;*
- *Directed edges connect the nodes two by two: Hence, if an edge exists then the two variables are preferentially dependent;*
- *To each node* $X$ *we associate a conditional preferential table (CP-Table). The* $CPT(A)$ *gives a local preference rule* $(A, u :\succ)$ *for each combination of values* $u \in pa(A)$ *for the parents of* $A$*.*

To each dependent node, we associate a $CPT(A)$ describing the user preferences over the values of the variable given all the parents possible combinations. Note that, each attribute is conditionally preferentially independent given its parents. Independent nodes (without parents), have only one row conditional preferential tables associated to the order of preference order over its values.

In what follows, we are going to deal only with acyclic graphs. We should mention that nothing in the semantic forces the graph to be acyclic. Likewise, we assume here CP-nets with fully specified preferential conditional tables.

A CP-net is called satisfiable if there exists at least one ranking of preferences that satisfies it. Thus, the ranking has to satisfy each of the conditional preferences found in the CP-Tables using the Ceteris Paribus principle. Note that, every acyclic CP-net is satisfiable.

We illustrate the CP-nets semantics with the next example.

**Example 1.1.** *Let us consider the simple CP-net of Figure 1.1 that expresses a user preferences over housing configurations. This network consists in 3 variables T, L and P, standing for the type, the locality and having pets respectively s.t* $D(T) = \{villa(v), flat(f)\}$*,* $D(L) = \{outskirt(o), downtown(d)\}$*,* $D(P) = \{dog(b), cat(c)\}$*.*
*The preference conditional set is:*

| |
|---|
| The user prefers having a villa to having a flat. |
| If he has a villa, he prefers it to be in the outskirt than in downtown. |
| If he has a villa, he prefers to have a dog to a cat. |
| If he has a flat, he prefers it to be downtown than in the outskirt. |
| If he has a flat, he prefers to have a cat to a dog. |

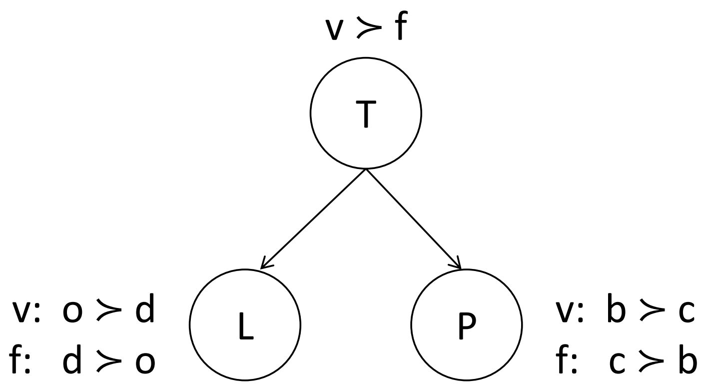



Figure 1.1: CP-net representation

*The universe of discourse associated to this example is* $\Omega = \{\omega_1 = voc,\ \omega_2 = vob,\ \omega_3 = vdc,\ \omega_4 = vdb,\ \omega_5 = foc,\ \omega_6 = fob,\ \omega_7 = fdc,\ \omega_8 = fdb\}$.
*Here, the node 'L' standing for locality is conditionally dependent on the node 'T'. So, we find to different orderings between the values of 'L' according to the parent configuration. For example, for T=v we have* $o \succ d$.

In the CP-net's semantics, parents priorities are more important than children's ones. Consequently, violating a father constraint is more important than violating a child one. After presenting the main characteristic of the CP-net, in the next section, we are going to present its principal semantics.

## 1.5 Optimization and ordering queries

The main motivation of the CP-nets is to have a procedure for combining the elementary evaluations to design a preorder between the alternative choices. Intuitively two principal queries may be asked. First, finding the optimal outcome, called optimization query. Second, which is more complex, order all the outcomes. In the following, the two types of queries are presented.

### 1.5.1 Outcome optimization

Finding the optimal outcome consists on finding a variable configuration where all the constraints (preferences) are satisfied. In the acyclic CP-net this procedure is straightforward while it is is not the case with cyclic graphs where answering this query needs an NP-hard algorithm. As already mentioned, acyclic CP-nets induce a unique optimal outcome. It can be found by a simple sweeping through procedure in a linear time and we assign each time the most preferred value according to the parents configuration. More precisely: To find the optimal outcome two different steps should be done:

1. First, choose for all the root nodes (independent variables) of the CP-net the most preferred values which are ordered first relatively to all the other domain values.
2. Then, each time when all the parents of a variable are assigned, choose its most preferred values according to the parents configuration. Repeat this step until all the variables are assigned.

The resulting outcome of this procedure is the optimal outcome. Let us show the application of this procedure on Example 1.1.

**Example 1.2.** *Applying the sweeping forward procedure on the Example 1.1:*

- *At first, we choose the value of the independent variable* $'T'$. *Thus we have* $T = v$.
- *Then we can process and choose the values of the two variables* $'L'$ *and* $'P'$. *We will have* $L = o$ *and* $P = b$.

### 1.5.2 Outcome comparison

Another task supported by the CP-nets is the comparison between the different alternatives. This query is more complex than the first one. Three possible alternatives between two interpretations exist. Consider two outcomes $\omega_1$ and $\omega_2$ the three possible relations are:

- The CP-net entails that $\omega_1$ dominates $\omega_2$;
- The CP-net entails that $\omega_2$ dominates $\omega_1$;

- Or, both of the outcomes are not comparable. It means that the CP-net does not contain enough information to determine that either outcome is preferred than another.

Starting with the dominance query (Boutilier et al., 1999), given two different outcomes $\omega_1$, $\omega_2$, a dominance relation is a precise relation of preference holds. For example, $\omega_1$ dominates $\omega_2$. The other way of comparison is just to prove that these outcomes are comparable. Therefore, a dominance relation is just to determine if a relation between them exists and it does not matter either of the outcomes dominates the other. It is clear that the ordering queries are weaker than the dominance queries.

Ordering queries in the acyclic graphs can be answered in a linear time in the number of variables. The following algorithm presents how this can be done. Note that this procedure exploits the graphical component of the CP-net to determine the hierarchical structure of the variables.

**Definition 1.5.** *(Boutilier et al., 2004) Let us consider an acyclic CP-net with N variables, and $\omega_1$ and $\omega_2$ be two outcomes. If there exists a variable $A$ in $V$, such that:*

1. *$\omega_1$ and $\omega_2$ assign the same values to all ancestors of $A$ in $V$;*
2. *Given the assignment provided by $\omega_1$ ($\omega_2$) to $pa(A)$ (such that $pa(A)$ is the set of all the parents nodes of the variable x), $\omega_1$ assigns a more preferred value to $A$ than that assigned by $\omega_2$.*

*Then $\omega_1 \succ \omega_2$.*

Based on the Ceteris Paribus semantics and using the information in the CP-Tables of the CP-nets, one can change or *flip* the value of a variable. This modification can lead either to an improved outcome or a worsened one. Therefore, we can partially order all the outcomes using this notion of swap pairs (Boutilier et al., 2004).

The set of ordered swap pairs is called a worsening tree where the root is the best outcome and the leaves are the worst ones. We can say that an outcome is preferred to another if there exists a chain (directed path) of worsening flips between them. Analogously, we can define an improving flipping sequence which is the reverse of the worsening sequence.

**Example 1.3.** *Let us consider the worsening tree in Figure 1.2 of the CP-net of Figure 1.1.*

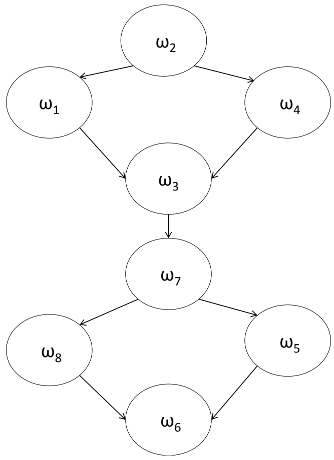



Figure 1.2: Worsening flip of Example 1

Consider the two outcomes $\omega_1$ and $\omega_4$, there is no directed path between them. Thus, the two outcomes are not comparable. We can also notice that there is only one value change between two outcomes related with an edge. Besides, from this ordering we can observe that the higher violations are (according to the CP-net graph) the larger is the negative impact on their order. But, we still can not compare two lower levels violation with a single ancestor preferences.

The complexity of the dominance testing in the binary valued variables depends directly on the structure of the CP-net graph. In particular:

- If it is a tree structured graph, the complexity of dominance testing is quadratic in the number of variables.
- When it has a polytree structure, it is polynomial in the size of the CP-net description.
- When a CP-net is singly connected and there is at most one directed path between any pair of nodes, the dominance is NP-Complete.

It was proved in (et al., 2008) that the temporal complexity of the dominace query in the general CP-nets (cyclic) is NPSPACE.
The flipping sequence search algorithm for tree structured graph was proposed by Brafmann (Boutilier et al., 2004). It is considered as the lower bound complexity algorithm. Assume we want to compare two interpretations $\omega_1$ and $\omega_2$. It starts first by assigning to all the variables the less preferred values according to the outcome. Then, iteratively, all the leaf variables will be removed. In one step, there will be no variable left, at that stage all the

variables will be assigned to their value of $\omega_1$. Finally, an improving flipping sequence from $\omega_1$ to $\omega_2$ will be generated.

In each step a candidate variable is chosen to be flipped. A variable is a candidate variable if:

- Its value can be flipped.
- There is no descendant of the variable where we can have its value flipped, knowing the current other variable values of the interpretation.

The corresponding algorithm is described in Algorithm 1.

**Algorithm 1** TreeDT ($[N \models \omega \succ \omega^{'}]$)

Begin.
Initialize the variables in $V$ to outcome $\omega^{'}$:
**while** *there is a non flipped value* **do**
    Iteratively remove all leaf variables from $V$ that have assigned to them their values in $\omega$.
    If $V = \emptyset$, then return yes.
    Find a variable $A$ s.t. its value can be improved, and no value of its descendants in $N$ can be improved, given the current assignment to $V$.
    If no such variable was found, then return no.
    Otherwise, change the value of $A$.
End.

## 1.6 Probabilistic CP-nets

Many extensions of the CP-net were proposed during this last decade namely TCP-nets (Brafman & Domshlak, 2002) and Probabilistic CP-nets (PCP-net for short) (Damien, Zanuttini, Fargier, & Mengin, 2013). TCP-net enables the processing of relative importance over the variables in the CP-nets. Therefore, they refine the order induced by the Ceteris Paribus constraints.

PCP-nets were briefly evoked in (de Amo, Bueno, Alves, & da Silva, 2012) for preference elicitation without giving a precise definition of their semantics. Then, they were deeply studied in (Damien et al., 2013). They were introduced to encounter the fact that real life scenarios are generally pervaded with uncertainty. One other feature of the PCP-nets, is their ability to describe compactly a set of person's preferences. Therefore, they are an aggregation or a summary of a group's preferences. In addition, PCP-nets afford the ability to answer about the probability of preference between two outcomes. Indeed, PCP-nets enable to compactly represent a probability distribution over many CP-nets and answer their associated queries. Probabilistic CP-nets can tolerate probabilities either on the conditional tables or on the edges either on both.

A PCP-net is a structure that has the same variables and domains as the CP-net, thus, the same graphical component. But instead of CP-Tables, they have PCP-Tables (as exemplified below). Also, each edge is weighted with a probability distribution.

**Example 1.4.** *This simple example presents the differences between PCP-nets and CP-nets:*
*event p is certain.*

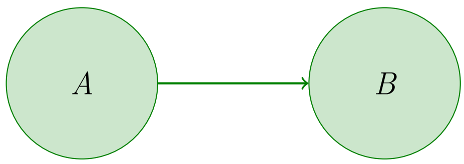



Table 1.1: Probabilistic preference conditional tables

| $P$ | ordering for $A$ |
|---|---|
| 0.8 | $a_1 \succ a_2$ |
| 0.2 | $a_2 \succ a_1$ |

| A | $P$ | ordering for B |
|---|---|---|
| $a_1$ | 0.3 | $b_1 \succ b_2$ |
| | 0.7 | $b_2 \succ b_1$ |
| $a_2$ | 0.2 | $b_1 \succ b_2$ |
| | 0.8 | $b_2 \succ b_1$ |

*For example, one possible interpretation of this PCP-net, there is 80% of people who prefer $a_1$ to $a_2$ and 20% who prefer $a_2$ to $a_1$.*

We have to mention that CP-Tables can be seen as PCP-Tables with a probability value associated to each preference equal to 1. Therefore, they are considered as a generalization or aggregation of CP-Tables.

From a PCP-net we can generate more than one CP-net, each one corresponding to a special choice performed to the PCP-net tables and edges. We can generate CP-Tables when choosing just one row for each value of a variable giving its parents. Generating CP-net we can find:

- CP-nets with the same graph structure;
- CP-nets with less edges;
- CP-nets with less nodes;
- or a combination of those alternatives.

Algorithms to determine the probability of the induced CP-net, the most probable CP-net and the optimal outcome are detailed in (Cornelio, Goldsmith, Mattei, Rossi, & Venable, 2013).

## 1.7 Conclusion

This chapter presents brief overview of a very efficient model for handling preferences and its ordering which reflects conditional dependence and independence of preference under the Ceteris Paribus principle. This formal framework often allows compact and arguably natural information representation. We argued about, given a CP-net, the basic inference problems and have proposed the different queries and their algorithms. We also mentioned that this representation can be used under the probabilistic framework to handle preferences pervaded with uncertainty.

Chapter 2

# Possibility theory: Preference representation approaches

## 2.1 Introduction

Non-classical theories were proposed in order to deal uncertain and imprecise information. We can cite the theory of evidence (Glenn Shafer, 1976) and the possibility theory (Dubois & Prade, 1988; Zadeh, 1978). Possibility theory presents an efficient tool to express knowledge and reason with it. It is, as well, very efficient to deal with preferences (Dubois et al., 2006; Kaci, Dubois, & Prade, 2004). One of its major aspects is its simplicity while being expressive.

A remarkable feature of the the possibility theory is the existence several representation formats. Indeed, the possibilistic bases (weighted propositional formulas), comparative bases (a set of strict comparative statements) and possibilistic networks where links are weighted by possibility degrees.

This chapter exhibits the definitions of these formats and their specifications and the possibility of translations between them. Section 2.2, and 2.3 proposes the basic concept of the possibility theory. Then, Section 2.4 tackles the different ways to express preference under the possibilistic framework.

## 2.2 Possibility theory

Possibility theory, introduced by Zadeh and developed by Prade and Dubois (Dubois & Prade, 1988; Zadeh, 1978), handles uncertainty in a qualitative way, while encoding it in the interval $[0, 1]$ called possibilistic scale.

### 2.2.1 Possibility distribution

Possibility theory (Dubois & Prade, 1988; Zadeh, 1978) relies on the idea of a possibility distribution $\pi$, which is a mapping from a universe of discourse $\Omega$ to a finite scale $L$. This possibility distribution is a function $\pi : \Omega \rightarrow L$.
The scale $L$ can be interpreted in two manners:

- Ordinal manner when all that matters is the order taking by the interpretations not the values they handle. Here, we talk about the qualitative setting. Thus, possibility degrees should take a value in the finite interval. Then, each value must be justified into the unit interval [0,1]. In most cases, it is difficult to attribute exact numerical values of possibility degrees. Therefore, it is easier for experts to say that one situation is more plausible than another.
- Numerical manner if the values associated to the interpretations have sense. This is called the quantitative setting. It is obtained by a finite and ordered scale denoted by $L = \{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n\}$ and $\alpha_1 = 1 > \alpha_2 > \alpha_3 > ... > \alpha_n = 0$ where each value makes sense.

**Example 2.1.** *Assume 3 values of a variable relative to 3 types of illnesses detected by a doctor i.e.; $\omega_1 = CANCER$, $\omega_2 = FLUE$ and $\omega_3 = AIDS$. After the diagnosis the doctor gave this possibility distribution:*
$\pi(CANCER) = \alpha_0$, $\pi(FLUE) = \alpha_5$, $\pi(AIDS) = \alpha_9$.
*From this possibility distribution $\pi$, we can deduce that CANCER is more plausible than the FLUE, which in its turn more plausible than the AIDS since $\alpha_0 = 1 > \alpha_5 = 0.5 > \alpha_9 = 0.1$.*

The definition of the possibility distribution for handling uncertainty is different from its meaning when talking about preferences. Either the two manner, the possibility distribution rank-order the possible states by their possibility level depending on the possible information. By convention:
For $\omega,\ \omega' \in \Omega$:

- $\pi(\omega) = 0$ means that the interpretation $\omega$ is fully impossible (totally refused).
- $\pi(\omega) = 1$ means that $\omega$ is totally possible (the most satisfactory). Nothing prevents it to be the real world.
- $\pi(\omega) > \pi(\omega^{'})$ means that $\omega$ is more plausible (preferable) than $\omega^{'}$.

The possibility distribution $\pi$ is normalized if $\exists\ \omega \in \Omega, \pi(\omega) = 1$, which expresses that there is at least one fully possible state in the possibility distribution. Therefore, not all values in $\Omega$ are impossible, and thus consistency. Note that nothing forbids to have $\omega \neq \omega^{'}$ and $\pi(\omega) = \pi(\omega') = 1$.

**Definition 2.1.** *Normalization: A possibility distribution $\pi$ is said to be $\alpha -$normalized, if its normalization degree, denoted by $h(\pi)$, is equal to $\alpha$, namely: $\alpha = h(\pi) = max_{\omega}\pi(\omega)$ when $\alpha = 1$ $\pi$, is said normal.*

**Example 2.2.** *Let us consider the same universe of discourse of Example 2.1. The possibility distribution is as follows: $\pi(\omega_1) = 0.2$, $\pi(\omega_2) = 1$, $\pi(\omega_3) = 0$.*
*This means that the most possible illness is the FLUE while the AIDS is totally rejected (impossible) and a slight possibility for the patient to be having Cancer.*
*Now consider:*
$\pi(CANCER) = 0$, $\pi(FLUE) = 1$, $\pi(AIDS) = 0$.
*He is sure that the disease is the FLUE.*
$\pi(CANCER) = 1$, $\pi(FLUE) = 1$, $\pi(AIDS) = 1$.
*He is totally ignorant.*

### 2.2.2 Possibility and Necessity measures

In probability theory, uncertain knowledge about any state $q$ is represented by a single probability measure P. This measure is dual i.e. we can deduce the probability degree assigned to $q$ from its complement $(\neg q)$: $P(q) = 1 - P(q)$. Furthermore, the term "it is probable that $q$" means that "not $q$ is not probable", on the other hand it is possible that $q$ it does not mean that it is not possible that not $q$. The possibility theory differs from probability theory by the use of a pair of dual set functions derived from the possibility distribution named possibility and necessity measures.

Any normalized possibility distribution can be associated with a possibility measure and a dual necessity measure. The two grading measures can be obtained using these two

mappings:
Let p be an event.

- *Possibility measure* (consistency):
  $\Pi(p) = max\,\{\pi(\omega) : \omega \in [p]\}$ evaluates to what extent $p$ can be entailed from the available information.

- *Necessity measure* (certainty):
  $N(p) = 1 - \Pi(\neg p)$ evaluates to what extent satisfying p is imperative.

The possibility measure estates the extent that the event $p$ is not inconsistent with the information while the necessity measure determines the certainty of an event. In fact, more the opposite event of $p$ is impossible more the event $p$ is possible. hence, necessity and possibility measure are two complementary measures. Yet, they satisfy those properties:

Let $p$ and $q$ be two different events then:

$$\Pi(p \vee q) = max(\Pi(p), \Pi(q)) \text{ and } \Pi(p \wedge q) = min(N(p), N(q)). \tag{2.1}$$

It is important to mention that the normalization of $\pi$ ensures that the possibility measure value according to an event is always lower than its necessity measure value. Giving $p$ an event, we can extract those possible states:

- $N(p) = \Pi(p) = 1$, p is certain and there no other possible information claiming the opposite.

- $N(p) > 0$ and $\Pi(p) = 1$, p is normally true. Almost all the possible situations are coherent with it.

- $N(p) = 0$ and $\Pi(p) = 1$, this is a state of complete ignorance about the certainty of the event $p$. In fact, both of $p$ and $\neg p$ are possible.

- $N(p) = 0$ and $\Pi(p) < 1$, $p$ is normally false.

- $N(p) = 0$ and $\Pi(p) = 1$, $p$ is certainly false.

**Example 2.3.** *Considering the universe of discourse* $\Omega = \{CANCER,\ FLUE,\ AIDS\}$
*Let* $\pi(\omega_1) = 0.2,\ \pi(\omega_2) = 1,\ \pi(\omega_3) = 0$ *be the possibility distribution.*
*Let p be "the patient suffers from CANCER or the FLUE". The corresponding measures to this event are:*

- $\Pi(p) = max(1, 0.2) = 1$.
- $N(p) = 1 - 0 = 1$.

*Thus, we can say that the event p is certain.*

### 2.2.3 Conditioning

Conditioning consists on modifying our initial knowledge about the state of the world, when a certain piece of information e arrives. In possibility theory, it is defined from the Bayesian-like equation $\Pi(A \cap B) = \Pi(A|B) * \Pi(B)$, where $*$ stands for the product in a quantitative setting (using the full power of the unit interval $[0,1]$), or for min in a qualitative setting where only the ordinal value of the grades makes sense.
The natural properties of possibilistic conditioning are:
Let $\phi = [e]$ be the set of models of e:

1. if $\pi(\omega) = 0$ then $\pi^{'}(\omega) = 0$;
2. $\forall \omega \notin \phi, \pi^{'}(\omega) = 0$;
3. $\pi^{'}$ should be normalized;
4. $\forall \omega_1, \omega_2 \in \phi, \pi(\omega_1) > \pi(\omega_2)$ iff $\pi^{'}(\omega_1) > \pi^{'}(\omega_2)$;
5. if $\Pi(\phi) = 1$, then $\forall \omega \in \phi, \pi^{'}(\omega) = \pi(\omega)$.

(1) means that impossible states stay impossible even after conditioning. (2) says that the information is totally certain. Then, (4) precise that the new information do not alter the order between the interpretations in $\phi$.

The above properties do not allow a unique conditioning definition. Indeed, two different conditionings in two different settings satisfy those properties namely in the ordinal scale and in the numerical scale.

- **In the ordinal setting:** the maximal possibility level is assigned to the best models of $p$. We get:

$$\pi(\omega|_m p) \begin{cases} 1 & \text{if } \pi(\omega) = \Pi(p), \omega \in [p], \\ \pi(\omega) & \text{if } \pi(\omega) < \Pi(p), \omega \in [p] \\ 0 & \text{if } \omega \notin [p]. \end{cases} \quad (2.2)$$

- **In the numerical setting:** the model of $p$ are proportionally shifted. We get:

$$\pi(\omega|_* p) \begin{cases} \frac{\pi(\omega)}{\Pi(p)} & \text{if } \omega \in [p], \\ 0 & otherwise. \end{cases} \tag{2.3}$$

**Example 2.4.** *Let us consider the universe of discourse and the possibility distribution of Example 2.1. Let p be "the patient suffers from CANCER or the FLUE". The corresponding possibility measure to this event is* $\Pi(p) = max(1, 0.2) = 1$.

- *Min-based:* $\pi(\omega_1|_m p) = 0.2$, $\pi(\omega_2|_m p) = 1$, $\pi(\omega_3|_m p) = 0$.
- *Product-based:* $\pi(\omega_1|_* p) = 0.2$, $\pi(\omega_2|_* p) = 1$, $\pi(\omega_3|_* p) = 0$.

## 2.3 Possibilistic networks

Possibilistic networks (Benferhat, Dubois, Garcia, & Prade, 2002; Ben Amor, Benferhat, & Mellouli, 2003; Benferhat & Smaoui, 2007) are a graphical compact format of the possibility theory and is the counterpart of the Bayesian networks (Pearl, 1985). They share the same basic components, namely:

- A *graphical component* which is a DAG (Directed Acyclic Graph) $\mathcal{G} = (V, E)$ where $V$ is a set of nodes representing variables and $E$ a set of edges encoding conditional (in)dependencies between them.
- A *numerical component* associating a local normalized conditional possibility distribution to each variable $A_i \in V$ in the context of its parents (denoted by $pa(A_i)$). The two definitions of possibilistic conditioning lead to two variants of possibilistic networks: in the numerical context, we get product-based networks, while in the ordinal context, we get min-based networks (also known as qualitative possibilistic networks).

Possibilistic networks are based on the idea of decomposing a joint possibility distribution as a combination of conditional possibility distributions. The procedure is similar to the one in probability theory. Hereafter, normalization constraints on each variable should be respected:

- If $pa(A_i) =$ so, for independent nodes we have $max(\Pi(A_i), \Pi(\neg A_i)) = 1$;

- For each dependent node $A_i$, having $pa(A_i) = \{u_i, \ldots, u_n\}$ be the set of the parents configurations, we have $max(\Pi(A_i|u_i), \Pi(\neg A_i|u_i)) = 1$.

Given a possibilistic network over $V = \{A_1, A_2, \ldots, A_N\}$ variables, we can compute its encoded joint possibility distribution using the following chain rule:

$$\pi(A_1, \ldots, A_N) = \otimes_{i=1..N} \quad \Pi(A_i \mid pa(A_i)). \tag{2.4}$$

Where $\otimes$ is either the *min* or the *product* operator $*$ depending on the semantic underlying it.

Thus, using for instance the min-based conditioning, we have :

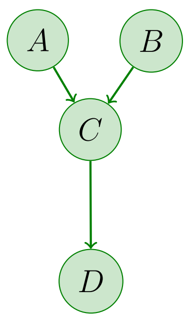
$$\pi(A_1,...,A_n) = \min(\pi(A_n|A_1,.., A_{n-1}), \pi(A_1,...,A_{n-1})). \tag{2.5}$$

When applying repeatedly this definition, the joint possibility distribution is decomposed into $\pi(A_1, ..., A_n) = \min(\pi(A_n|A_1, ..., A_{n-1}), ..., \pi(A_2|A_1), \pi(A_1))$

The conditional possibility distribution $\pi(A_i|A_1, ..., A_{i-1})$ associated with each variable $A_i$ can always be normalized by construction. Like for Bayesian networks, the above decomposition can be further simplified by assuming conditional independence between variables (Ben Amor & Benferhat, 2005). For instance, if $A_n$ is independent of $A_1, ..., A_i$ in the context $A_{i+1}, ..., A_{n-1}$ then the expression $\pi(A_n|A_1, ..., A_{n-1})$ can be simplified into $\pi(A_n|A_{i+1}, ..., A_{n-1})$.

**Example 2.5.** *Let us consider the following possibilistic network.*

| $\pi(a)$ | $\pi(\neg a)$ |
|---|---|
| 1 | 0.2 |



| $\pi(b)$ | $\pi(\neg a)$ |
|---|---|
| 1 | 0.8 |

| $\pi(.\|.)$ | $ab$ | $a\neg b$ | $\neg ab$ | $\neg a \neg b$ |
|---|---|---|---|---|
| $c$ | 1 | 1 | 1 | 1 |
| $c$ | 0.7 | 1 | 1 | 0.6 |

| $\pi(.\|.)$ | $c$ | $\neg c$ |
|---|---|---|
| $d$ | 1 | 0.2 |
| $\neg d$ | 0.3 | 1 |

Figure 2.1: A possibilistic network

*If we consider this graph as min-based then the corresponding joint distribution of the interpretation* $\omega = \neg ab\neg cd$ *is* $\pi(\neg a \vee b \vee \neg c \vee d) = min(\pi(\neg a), \pi(b), \pi(c|\neg ab), \pi(d|\neg c)) = min(0.2, 1, 1, 0.2) = 0.2$.
*If it is a product-based network then* $\pi(\neg a \vee b) = (\pi(\neg a) * \pi(b) * \pi(c|\neg ab) * \pi(d|\neg c)) = 0.04$.

## 2.4 Preference representation

Several methods aiming to represent preferences in the possibilistic framework exist, some are based on graph structures and others on logic theories (Benferhat et al., 1999). We can cite in particular, possibilistic logic, the comparative bases and symbolic possibilistic bases (Kaci & Prade, 2008; Dubois et al., 2006, 2013b; Benferhat et al., 1999). When this theory is used, possibility degrees will provide an information about how much satisfactory the alternative choices are. Thus, the possibility distribution will be restricted to alternative choices that are somewhat acceptable. Then, the possibility measure will be able to compute the satisfaction degree . $\Pi(\omega)$ estimates to what extent the user will be satisfied while $N(\omega)$ expresses the imperativeness of having the interpretation $\omega$. Besides, possibility theory has a very useful feature. Since, it can be be represented in many different formats. Also, it has the machinery to go from one format to another in a rather direct way.

The efficiency of one format changes with consideration on how the information was expressed. therefore, in some cases we can find formats that are more compact and concise than others although that they express almost the same thing. This is due to how it is natural for the user to express his preferences. Moreover, it is possible to use more than one format to express them (Benferhat & Smaoui, 2007). We mention in particular, comparative bases possibilistic bases and possibilistic networks, detailed below.

### 2.4.1 Comparative bases

A strict comparative possibilistic base is a set of constraints of the form $\Pi(v) > \Pi(f)$, it means that there is at least one interpretation where we have $v$ that is more preferred than one interpretation having $f$. This is considered a weak relation, since due to the use o the possibility measure, we consider that the most satisfactory interpretation having $v$ is preferred to the most preferred interpretation having $f$. For instance, 'when having a villa, I prefer to live in the outskirt than downtown'. It is the translation of the of the default rules $v \to d$ is preferred to $v \to o$.

We consider this format as the most direct one to represent preferences (Benferhat et al., 1999). It consists on simply strict comparisons between interpretations which seems to be the most natural way for a person to define his preferences. Thus, it is a proper preference representation framework.

**Example 2.6.** *Assume that we want to represent the preferences mentioned in Example 1.1 in the possibilistic framework. The most natural format to be translated into is the comparative (conditional) base. We have 5 constraints:*
$\Pi(v) > \Pi(f)$.
$\Pi(v \wedge d) > \Pi(v \wedge o)$.
$\Pi(f \wedge o) > \Pi(f \wedge d)$.
$\Pi(v \wedge b) > \Pi(v \wedge c)$.
$\Pi(f \wedge c) > \Pi(f \wedge b)$.
*For instance, when having a villa (v) the user prefers it to be outskirt (o) than downtown (d).*

Benfarfat et al. (Benferhat, Dubois, & Prade, 1992) proposed an algorithm to induce a unique qualitative possibility distribution from the strict comparative base. To rank the different interpretations, we have to split them into different strats where we have in the first partition interpretations that do not violate any of the constraints and so on. Applying the algorithm in (Benferhat, Dubois, Kaci, & Prade, 2001), it has been shown that interpretations can be classified into different partitions. The first one is the most satisfiable and the last one is the least one. It consists in, each time, putting interpretations in the highest possibility rank. This is an application of the Minimum of specificity principle i.e. anything not observed as actually possible, or asserted actually satisfactory is ruled out.

Yet, any alternative is by default assumed as much satisfactory as no other constraint assume it to be less satisfactory. Here, we recall the main steps of the algorithm in (Benferhat et al., 2001; Benferhat & Garcia, 1997). Consider the comparative formula $R_1 > L_2$, we put respectively in the set $i_1$ and $i_2$, alternatives that are satisfied by the corresponding logical formula. Then choose in each step the interpretations that are are not existing in any right set. The first layer will have the highest satisfaction degree and so on. The following algorithm describes the steps of the transformation to the set of well ordered partitions. Let $WOP(\pi) = S_1, S_2, \ldots, S_i$ be the set of ordered partitions such that $\Omega = S_1 \cap S_2 \cap \ldots, S_i$.

**Example 2.7.** *Considering the comparative base of Example 2.6, We obtain* 3 *different*

**Algorithm 2** Transformation from comparative base to set of ordered partitions

**Data**: Universe of discourse $\Omega$, counter i, the set of rules $G$

**Result**: WOP of sets $S_i$

$i \leftarrow 0 \;\; U \leftarrow \Omega$

**while** $U \neq \emptyset$ **do**

  $i \leftarrow i + 1$;

  $S_i = \omega : \not\exists\, \Pi(A_k \cap B_k) > \Pi(A_k \cap B_Z)$ s.t. $\omega \in A_k \cap B_k$;

  **if** $S_i = 0$ **then**

    G is inconsistent

  Remove from $U$ elements of $S_i$ Remove from G the constraints such that $S_i \cap A_k \cap B_k \neq \emptyset$;

return $WOP = E_1, E_1, \ldots, E_1$ s.t. $\forall j \leq i, E_j = S_{i-j+1}$;

End.

*layers:*
$E_1 = \{\omega_2\}$.
$E_2 = \{\omega_1, \omega_3, \omega_4, \omega_7\}$.
$E_3 = \{\omega_5, \omega_6, \omega_8\}$.

### 2.4.2 Possibilistic logic bases

Possibilistic bases (Dubois, Lang, & Prade, 1994) are sets of a finite propositional language $\phi_i$ (first-order logic) denoted by $\Sigma$. Each formula has a weight $\alpha_i$, belonging to the scale $[0, 1]$, which expresses to what extent the constraint is imperative in the preference settings with consideration to the incomplete available information. The possibilistic weight reflects how much each choice is satisfactory.

A N-possibilistic base is a possibilistic base where the weights are computed as the necessity measure. The possibilistic base is under the form $\Sigma = (\phi_i, \alpha_i) : i = \{1, \ldots, n\}$.

In the qualitative setting, this base can be presented as a set of partitions where each one contains formulas with the same necessity degree. The order between the alternatives is given by a possibility distribution. Thus, from this knowledge base we can generate a unique possibility distribution by associating to each interpretation the level of compatibility with the preference afforded:

$$\forall \omega \in \Omega, \pi_\Sigma(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i, \alpha_i) \text{ in } \Sigma,\; \omega \in [\phi_i] \\ 1 - max\{\alpha_i(\phi_i, \alpha_i)\} \in \Sigma \text{ and } \omega \notin [\phi_i] & \text{otherwise} \end{cases} \tag{2.6}$$

Therefore, the interpretations satisfying all the preference constraints will have the highest possibility degree 1. The converse transformation, from a possibility distribution to possibilistic base is also straightforward.

**Example 2.8.** *Let* $\Omega = \{\omega_1 = voc,\ \omega_2 = vob,\ \omega_3 = vdc,\ \omega_4 = vdb,\ \omega_5 = foc,\ \omega_6 = fob,\ \omega_7 = fdc,\ \omega_8 = fdb\}$ *be the set of interpretations.*
*The comparative format (Conditional base) could be translated into a possibilistic logic base (Benferhat et al., 2001) conserving the same possibility distribution. The corresponding possibilistic logic is then:*
$\Sigma = \{(f \vee o, 1/3), (f \vee b, 1/3), (v, 1/3), (v \vee d, 2/3), (v \vee c, 2/3)\}$.
*Given this possibilistic base we can generate a unique possibility distribution:*
$\pi(\omega_2) = 1, \pi(\omega_1) = \pi(\omega_3) = \pi(\omega_4) = \pi(\omega_7) = 2/3, \pi(\omega_5) = \pi(\omega_6) = \pi(\omega_8) = 1/3$.
*For example,* $\omega_7$ *violates* $(v, 1/3)$*. The satisfaction degree of* $\omega_7$ *is then* $\pi(\omega_7) = 1 - 1/3 = 2/3$.

Note that the possibility theory leads to a total ordering of all the alternative choices. The optimal outcome is the interpretation with the highest distribution degree. For instance, in Example 2.8, $\omega_2$ is the best outcome.

### 2.4.3 Symbolic possibilistic logic

Symbolic possibilistic bases (HadjAli, Kaci, & Prade, 2008) consist on adding to each formula a symbolic weight. The set of weights are going to set a partial ordering between the alternative choices. It was proved in (HadjAli et al., 2008) that this approach is more faithful to the user preferences than the CP-net. Ordering the outcomes could be done by the use either the Leximin or the Symmetric Pareto ordering. The former refines the latter. It was proved also that both of these ordering are considered to be the upper and the lower bound respectively

The definition of symbolic possibilistic logic is exactly the same as the possibilistic logic. To each entry of the form $u_i : x_i > \neg x_i$ in the conditional table of the CP-net of each node is encoded by the possibilistic logic clause $(\neg u_i \vee x_i, \alpha_i)$ where $\alpha_i$ is a symbolic weight. All the formulas created are put in the symbolic base $\Sigma$, such that, each node will have a single symbolic weight. Therefore, we can do the conjunction of all the formulas of one node. Note that we can add additional constraints over symbolic weights.

As mentioned above, we can generate a partial preoder of the alternative choices. Let $w = (w_1, ..., w_m)$ and $w' = (w'_1, ..., w'_m)$ be two weight vectors having the same number of

components ($w$ and $w'$ are viewed as sets). To each interpretation is associated a vector of weights such that if a constraint is satisfied than the corresponding value will be equal to 1 otherwise it will be equal to $1 - \langle$the constraint priority weight$\rangle$.
Formal definitions of Leximin and Symmetric Pareto are:

**Definition 2.2.** *Pareto:* $w \succ_{Pareto} w^{'}$ *iff* $\forall i, w_i \geq w_i^{'}$ *and* $\exists j$ *s.t.* $w_j > w_j^{'}$. $w \succ_{Pareto} w'^{\sigma}$ *iff* $min(w \cup w') \subseteq w'$.

**Definition 2.3.** *Leximin: First, delete all pairs* $(w_i, w_i')$ *from* $w$ *and* $w'$ *such that* $w_i = w_i'$. *Thus we get two non-overlapping vectors. Then* $w \succ_{leximin} w'$ *iff* $min(w \cup w') \subseteq w'$.

**Definition 2.4.** *Symmetric Pareto:* $w \succ_{SP} w'$ *iff there exists a permutation* $\sigma$ *of the components of* $w'$, *yielding a vector* $w'^{\sigma}$, *s.t.*[1] $w \succ_{Pareto} w'^{\sigma}$ *iff* $min(w \cup w') \subseteq w'$.

**Example 2.9.** *Let us consider the preferences mentioned in Example 1.1. The corresponding symbolic base is :* $k = \{(v, 1-\alpha), ((f \vee b) \wedge (v \vee c), 1-\beta), ((f \vee o) \wedge (v \vee d), 1-\delta)\}$. *Table 2 recapitulates the whole interpretations.*

Table 2.1: Symbolic possibilistic base

| $\Omega$ | $(v, 1-\alpha)$ | $((f \vee b) \wedge (v \vee c), 1-\beta)$ | $((f \vee o) \wedge (v \vee d), 1-\delta)$ |
|---|---|---|---|
| $\omega_1$ | 1 | $1-\beta$ | 1 |
| $\omega_2$ | 1 | 1 | 1 |
| $\omega_3$ | 1 | $1-\beta$ | $1-\delta$ |
| $\omega_4$ | 1 | 1 | $1-\delta$ |
| $\omega_5$ | $1-\alpha$ | 1 | $1-\delta$ |
| $\omega_6$ | $1-\alpha$ | $1-\beta$ | $1-\delta$ |
| $\omega_7$ | $1-\alpha$ | 1 | 1 |
| $\omega_8$ | $1-\alpha$ | $1-\beta$ | 1 |

Since we may not know to what extent preferences are imperative, the weights associated to the logical formulas are given as symbols. Still, they are supposed to belong to an ordered scale, and if some ordering is known between some of the weights,this information may be added. For instance, in the CP-net spirit, we may want in Example 2.9 that the parents weights are greater than the children's ones i.e. $1 - \alpha > \min(\beta, \delta)$. Even if the weights are expressed in terms of symbols, we can generate a pre-order using the Leximin ordering or the Symmetric Pareto orderings. Here we have the same pre-order: assuming $\alpha < \max(\beta, \delta)$, $\omega_2 \succ \omega_1 \sim \omega_4 \succ \omega_3 \succ \omega_7 \succ \omega_5 \sim \omega_8 \succ \omega_6$.

[1] $w \succ_{Pareto} w^{'}$ iff $\forall i, w_i \geq w_i^{'}$ and $\exists j$ s.t. $w_j > w_j^{'}$.

## 2.5 Conclusion

In this chapter, we surveyed contributions in the preference representations under the possibilistic framework that have been developed in the last decade. The possibility theory appears to be rich in representation formats, permitting to express many kinds of preferences. We pointed that the possibilistic representation setting allows for different representation formats that are all equivalent to the possibilistic logic format, but which may of interest depending on the way people express their preferences. One may distinguish the possibilistic logic, symbolic possibilistic logic and the comparative base. In the next chapter, we will propose a comparison between the different approaches of expressing the preferences.

Chapter 3

# Possibilistic preference modeling approaches vs CP-nets

## 3.1 Introduction

As presented in Chapter 1 and 2, there exist several methods aiming at representing preferences in the possibilistic framework. Some are based on graphical structures providing a compact way to express preferences and others on logic theories. In spite of the popularity and success of the CP-nets, present some limitations. Indeed, they may express more than what the user wanted to communicate. In this chapter, we provide a comparative study between orderings deduced from different possibilistic methods of preference representation and the CP-nets.

Generally, preferences, especially when the number of alternatives is huge, are not directly expressed by a total ordering. Rather, the decision maker describes his preferences locally, i.e., he only mentions features of interest and his preferences among them. Note that preferences can be expressed either by means of strict comparisons, or by prioritized goals. Thus, we need completion principles to induce a set of ordered interpretations. Each method presented in the previous chapters is based on a particular completion principle.

The possibilistic theory setting offers different representation formats (sets of weighted logical formulas, sets of conditionals, possibilistic networks) (Dubois & Prade, 2008), and different completion principles may be considered for obtaining an ordering from the preference specifications. Therefore, the differences and similarities between them merit a rigorous examination.

This chapter proposes a synthetic comparison between the methods of preference representation under the possibilistic framework and the CP-nets. We will focus on the expressive power of each method. Our comparative study will be based on how well do each method use the information given to induce a general comparison between alternatives.

Firstly, orders induced by different methods should be compared to the inclusion ordering presented in the following. Second, we will discuss how does each method react to the violated constraints and what are the completion principles used. Do they lead to total or partial pre-orders? We identify the positive aspects and the limitations in each case.

This chapter is partitioned following this outline. Section 3.2 presents the Pareto ordering. Section 3.3, 3.4 and 3.5 discuss the different orderings of the methods presented in Chapter 1 and 2. Finally, Section 3.6 provides a synthetic discussion.

Principle results of this chapter are accepted to be published in (Ben Amor, Dubois, Gouider, & Prade, 2014b).

## 3.2 Pareto ordering

Since in many practical situations, ranking the alternatives according to a finite numerical scale is impossible and too complex to be used, other solutions should be studied. Indeed, this problem amounts to compare those alternatives without aggregating them. The problem of ranking the interpretations consists on, given a set of constraints $C = \{s_1, \ldots, s_i\}$ of the form $a \succ b$ in the context $c$, is to compute an order with no added supplementary constraints. The natural way for ranking the alternatives is the well-known Pareto ordering, which is:

**Definition 3.1.** *Pareto: Let $v$ and $v'$ be two vectors. $v \succ v'$ iff $\forall i,\ v_i \geq v_i'$ and $\exists\ j$ s.t.$v_j > v_j'$.*

**Example 3.1.** *(Boutilier et al., 2004) Let us consider the following preferences. We assume that all the variables are binary for the sake of simplicity. This example is going to be used for the rest of the chapter. It expresses preferences about housing configurations over 3 variables standing for the main course ($M$), the soup ($S$) and the drink ($W$) s.t. $D(M) = \{meatcourse(M_{mc}), fishcourse(M_{fc})\}$, $D(S) = \{fishsoup(S_f), vegetablesoup(S_v)\}$, $D(W) = \{redwine(W_r), whitewine(W_w)\}$.*
*The preference conditional set is:*

*The user prefers a meat course ($M_{mc}$) to a fish course ($M_{fc}$)*
*If the main course is meat, he prefers to have a fish soup ($S_f$) to a vegetable one ($S_v$)*
*If the main course is fish, he prefers to have a vegetable soup to a fish soup*
*If the is served a vegetable soup, he prefers to have red wine ($W_r$) to white one ($W_w$)*
*If the is served a fish soup, he prefers to have white wine to red one*

*The universe of discourse associated to this example is* $\Omega = \{\omega_1 = M_{mc}S_fW_w,\ \omega_2 = M_{mc}S_fW_r, \omega_3 = M_{mc}S_vW_w,\ \omega_4 = M_{mc}S_vW_r,\ \omega_5 = M_{fc}S_fW_w,\ \omega_6 = M_{fc}S_fW_r,\ \omega_7 = M_{fc}S_vW_w,\ \omega_8 = M_{fc}S_vW_r\}$.

To each interpretation we associate a vector of constraints, we assign 1 to each violated constraint $s_i$ otherwise 0. Then the comparison is done according to the sum of the vector values. For instance, Let $v_{\omega_1} = \{0,0,0,0,0\}$ and $v_{\omega_6} = \{1,0,1,0,1\}$ be two interpretation vectors, $\omega_1$ is preferred to $\omega_6$. The corresponding order of Example 3.1 is: $\omega_1 \succ \omega_2 \sim \omega_4 \sim \omega_8 \succ \omega_3 \sim \omega_5 \sim \omega_7 \succ \omega_6$. Note that the highest violated constraint number is 3. We can not violated more than this number due the conditioned statement of the constraints. Pareto Ranking is the most natural way as it takes into consideration the number of local preferences violated in the sense that if an interpretation $v$ satisfies all the constraints satisfied by another interpretation $v'$ plus some other(s), then $v$ is strictly preferred to $v'$.

Our unique criteria here is the number of the violations. Therefore, inducing this order is straightforward. Besides, it does not take into consideration the relative importance between the variables or any eventual interaction and dependencies between them.

Our first goal is to compare the different orderings with the inclusion order with respect to the number of violated preferences. All the induced orders must be faithful to it otherwise they would be inconsistent. Yet, Pareto ordering is not sufficient to represent preferences. Hereafter, it should be refined by introducing some completion principles. As mentioned above, each method presented has a completion principle namely the maximum and the minimum of specificity, the Ceteris Paribus and the leximin ordering.

## 3.3 CP-net ordering

CP-net, directed graph expressing conditional statements, is based on the *Ceteris Paribus* principle (which sounds very intuitive). However, even improved versions of CP-nets, such as TCP-nets (Brafman & Domshlak, 2002) that take into account the relative importance between variables, still suffer from noticeable limitations.

During the comparison we are going to rely on the three elementary structures of CP-net. Any CP-Net graph should be a combination of these structures with perhaps more parents or children nodes. Thus, given those structures we can gather all the possible situations.

**Example 3.2.** *(Boutilier et al., 2004) Let us consider the following CP-nets. We assume that all the variables are binary for the sake of simplicity. This example is going to be used for the rest of the chapter. Figure 3.1 illustrates the corresponding CP-nets such that:*

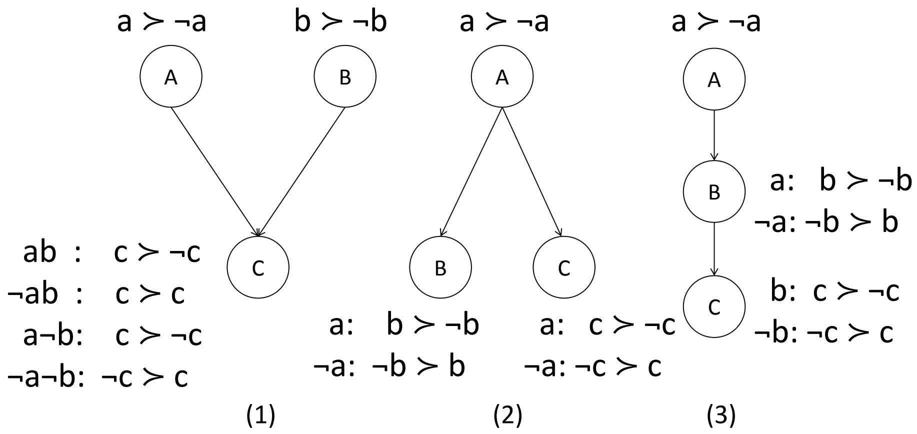



Figure 3.1: Elementary structures of CP-Net

*The associated universe of discourse is* $\Omega = \{\omega_1 = abc,\ \omega_2 = ab\neg c,\ \omega_3 = a\neg bc,\ \omega_4 = a\neg b\neg c,\ \omega_5 = \neg abc,\ \omega_6 = \neg ab\neg c,\ \omega_7 = \neg a\neg bc,\ \omega_8 = \neg a\neg b\neg c\}$.

These three structures illustrate the following three cases:

- Case(1): Two parent nodes with one child node.
- Case(2): One parent node with two children nodes.
- Case(3): Each parent have one child. A parent node with a grandchild node.

The ordering can be determined using the worsening flip method, based on the Ceteris Paribus principle and illustrated by Figure 3.2. The orderings deduced from the CP-nets are partial, many interpretations are not comparable, and it induces only one optimal

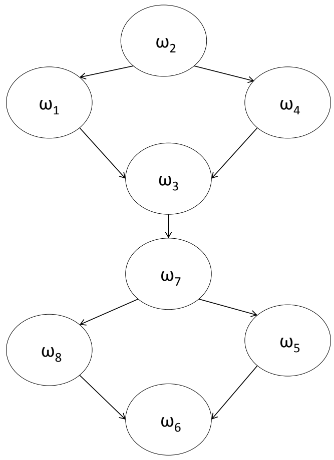



Figure 3.2: Worsening flip of case (2)

(worst) outcome. For instance, $\omega_8$ and $\omega_3$ are not incomparable since no directed path exists between them. Note that those two interpretations were compared using the Pareto ordering $\omega_8 \succ \omega_3$. It is also worth mentioning that the CP-nets are faithful to the Pareto ordering although that no proof has been proposed yet (either a counter-example).

The systematic application of Ceteris Paribus, used as a completion principle for building a partial order from the specification of conditional preferences, induces some additional information (such as apparent priorities between preferences) that is not explicitly stated in the preference description (Dubois, Prade, & Touazi, 2013a). For this purpose, let us consider the following example:

**Example 3.3.** *Let $V = \{A, B, C, D, E\}$ be the set of binary variables and $\Omega = \{\omega_1 = acb\neg d\neg e,\ \omega_2 = a\neg c\neg b\neg d\neg e,\ \omega_3 = \neg a\neg c\neg b\neg d\neg e,\ \omega_4 = ac\neg b\neg d\neg e\}$ be the set of the alternative choices. The order induced by the CP-net is $\omega_1 \succ \omega_2 \succ \omega_3$ and $\omega_1 \succ \omega_4$. Note that this order leave way to non comparable interpretations. In fact, it can not decide about the preference over $\omega_4$ and $\omega_4$ nor $\omega_3$ and $\omega_4$.*

Two limitations should be highlighted:

- Priority: In the CP-net approach the priority over the variables is totally and somewhat rigidly given through the structure of its graph. In fact, importance is assigned to parents nodes relatively to their children which is not evident in the preference setting. Precisely, the forced priority in favor of father nodes with respect to child

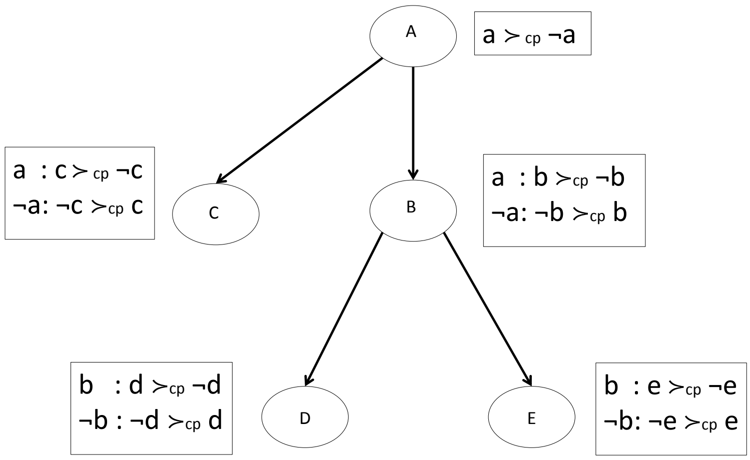



Figure 3.3: CP-net of Example 3

nodes. Besides, this kind of automatic priority is given locally and this is the subject of the next limitation.

- Transitivity of priority: We noticed that falsifying a grandchild preference is better than falsifying a child one. And this latter is as well better than falsifying a parent preference ($\omega_1 \succ \omega_2 \succ \omega_3$). This is the direct application of the semantics held implicitly by the CP-net. However, this implicit principle is not always satisfied since there is non possible comparison between two child preferences ($\omega_2$) and with falsifying one child and one grandchild preferences ($\omega_4$). Even more, it can not compare one parent violation ($\omega_3$) and one child and one grandchild preferences ($\omega_4$). Therefore, it is obvious to see that there is additional information, implicitly added, that blocks the transitivity of priority (Dubois, Prade, & Touazi, 2013c).

Finally, CP-nets are, in some sense, both too *bold* and too *cautious*. Too bold since, as a result of the systematic application of the Ceteris Paribus principle, some priority is given to preferences associated to parent nodes which cannot be questioned and modified, as already said. Too cautious since they usually lead to a partial order while a complete preorder may be more useful in practice. We saw also that CP-nets suffer from the lack of transitivity between the priorities. For instance, they do not acknowledge the fact that violating the preference of two grandchildren is less important than violating one child and one grandchild preferences. This is contradictory with the basic concept of CP-nets namely the father priority is more important than the child one.

Thus, the partial ordering induced by the CP-net approach may look somewhat debatable, as presented above, wondering the exactitude of the preference representation. This, may in turn questions the capability of the possibility framework to handle preferences in a more global manner than the CP-net's way.

## 3.4 Possibilistic base ordering

Possibilistic setting is another framework that can be used for preference representation. Beside its capability to express knowledge efficiently and reason with it, this logic is, as well, very effective to deal with preferences. It induces a total preorder thanks to its possibility distribution (Dubois et al., 2006). In this theory, we can distinguish two completion principles namely the minimal and maximal of specificity that respectively produce the largest and the smallest possibility distribution (Kaci & van der Torre, 2008; Dubois, Kaci, & Prade, 2005).

Obviously, the comparative base is the most direct way to describe preferences. Each local preference is translated into a strict comparison between two situations.

**Example 3.4.** *The preferences described in Example 3.1 are expressed in the following way:*
$\Pi(M_{mc}) > \Pi(M_{fc})$.
$\Pi(M_{mc} \wedge S_f) > \Pi(M_{mc} \wedge S_v)$.
$\Pi(M_{fc} \wedge S_v) > \Pi(M_{fc} \wedge S_f)$.
$\Pi(S_f \wedge W_w) > \Pi(S_f \wedge W_r)$.
$\Pi(S_v \wedge W_r) > \Pi(S_v \wedge W_w)$.

This order could be extracted directly from the comparative base using the minimum of specificity. It consists on considering interpretations, which are not less preferred than any other, to be totally satisfiable. Another opposite way of reasoning is the maximum of specificity, it consists in the fact that each interpretation dominated by any other interpretation to be the least satisfiable.

Indeed, the minimal specificity principle accord the most important degree to alternatives. It does not enforce any preference between the criteria if not explicitly provided. Contrariwise the maximal of specificity gives the lowest possible degree to the alternatives.

Note that both of the principles induce total preodrers with a number of levels less than the Pareto partial order with being definitely faithful to it. The following example illustrates the two completion principles.

In fact, they can refer to two kinds of decision makers. Optimistic ones according to the maximum of specificity, because they tend to give the interpretations the highest satisfactory degree 1, and cautious ones according to the minimum of specificity. Giving the highest degree only to interpretations that are never dominated by another. Another thing to specify is that all the interpretations are compared and there is no room for incomparable interpretations which is not the case with the inclusion order. We can look also for an eventual combination between them.

As mentioned above moving from one format to another can be done in an almost direct way. These same preferences are translated into a possibilistic base, under the form of prioritized goals. It is a simple hierarchy of goals.

**Example 3.5.** *The corresponding possibilistic base is then*
$\Sigma = \{(M_{mc}, 1/3),(M_{fc} \vee S_f, 1/3),(S_v \vee W_w, 1/3),(M_{mc} \vee S_v, 2/3),(S_f \vee W_r, 2/3)\}$.

Using $\Sigma$, we can generate a unique possibility distribution: $\pi(\omega_1) = 1,\ \pi(\omega_2) = \pi(\omega_4) = \pi(\omega_8) = 1/3,\ \pi(\omega_3) = \pi(\omega_5) = \pi(\omega_6) = \pi(\omega_7) = 2/3$.

From this possibilistic base we can generate a total preorder represented by the possibility distribution $\omega_1 \succ \omega_2 \sim \omega_4 \sim \omega_8 \succ \omega_3 \sim \omega_5 \sim \omega_7 \sim \omega_6$. It is important to point also that the possibilistic order is coherent with the Pareto ordering but we noticed that it tends to flatten it. In fact, it is due to the use of the minimum function as it considers only the most important constraint violated (the violated formula with the highest necessity value) and retains the worst satisfaction degree. Precisely, let the alternative $\omega$ be associated with a weights vector $u_{\omega_1} = (a_1, a_2, \ldots, a_i)$ where $a_i \in [0, 1]$ . The possibility degree $\pi(\omega) = x$ where $\forall a_i\ x \succ a_i$.

We can highlight two important limitations:

- No graphical component: The possibilistic logic for handling preference is very robust and it is constructed over strong basics. However, it is easy to notice the hardness of determining the dependent variables and this is due to the absence of a graphical component.
- Complete order: The order induced is even less precise than the Pareto ordering. Indeed the number of levels is strictly under the Pareto levels. Therefore, we can

not retain much information from it. Since the possibilistic ordering is complete, we notice that it generates equalities between the interpretations that are debatable.

Finally, the comparative constraints (or prioritized goals) produces exceedingly cautious orders. Such order intersect with the CP-net ordering. It is worth noticing that, using the Cetreris Paribus principle, we are unable to define default constraints. Indeed, in the CP-net, it is not possible to say that a preference hold except in a context $c$, which is totally possible using the possibilistic framework and more precisely the comparative base. For obvious examples, this may not cause a problem, but in more complex situations this may seem enough imposing.

## 3.5 Symbolic possibilistic base ordering

As mentioned above, possiblistic logic can code partial preorder when using symbolic weights. Therefore, each formula will have a symbolic degree expressing its priority among the others. Those symbolic weights can be ordered such that parents constraints have higher weights than children ones. To order the interpretations we can use either Leximin or Symmetric Pareto ordering. To each interpretation we associate a vector of weights. Each weight is equal to 1 if the corresponding constraint otherwise it will be equal to 1-(the highest priority weight).

**Definition 3.2.** *Leximin: First, delete all pairs* $(w_i, w'_i)$ *from* $w$ *and* $w'$ *such that* $w_i = w'_i$*. Thus we get two non-overlapping vectors. Then* $w \succ_{leximin} w'$ *iff* $min(w \cup w') \subseteq w'$*.*

**Definition 3.3.** *Symmetric Pareto:* $w \succ_{SP} w'$ *iff there exists a permutation* $\sigma$ *of the components of* $w'$*, yielding a vector* $w'^{\sigma}$*, s.t.*[1] $w \succ_{Pareto} w'^{\sigma}$ *iff* $min(w \cup w') \subseteq w'$*.*

The order induced is more respectful to these constraints than the CP-net ordering because it conserve the transitivity of priorities. Applying the Leximin ordering on the structure (3) of Example 3.1 we find this order is $\omega_1 \succ \omega_2, \omega_4, \omega_8 \succ \omega_3, \omega_5, \omega_7 \succ \omega_6$. Leximin and Symmetric Pareto are respectively the lower and the higher bound orderings of the CP-net. They generally only approximate that order (with being coherent with it). The only case where they both yield to the same order is when each parent node has at most one child.

[1] $w \succ_{Pareto} w'$ iff $\forall i, w_i \geq w'_i$ and $\exists j$ s.t. $w_j > w'_j$.

**Example 3.6.** *Let $V = \{a, b, c\}$ the set of variables used in Example 3.3. The symbolic possibilistic bases of the different preference examples (cases 1,2 and 3) are as follows:*

- $\Sigma_1 = \{(a, \alpha_1),\ (b, \alpha_2),\ (((\neg(a \wedge b) \wedge \neg(\neg a \wedge \neg b)) \vee c) \wedge (\neg(a \wedge \neg b) \wedge \neg(\neg a \wedge b))\neg c), \alpha_3)\}$, *with* $min(\alpha_1, \alpha_2 > \alpha_3$.
- $\Sigma_2 = \{(a, \alpha_1),\ ((\neg a \vee c) \wedge (a \neg c), \alpha_4),\ ((\neg a \wedge b) \vee (a \wedge \neg b, \alpha_5)\}$, *with* $\alpha_1 > Max(\alpha_4, \alpha_5)$.
- $\Sigma_3 = \{(a, \alpha_1),\ ((\neg a \wedge b) \vee (a \wedge \neg b, \alpha_5),\ ((\neg b \vee c) \wedge (b \vee \neg c), \alpha_6)\}$, *with* $\alpha_1 > \alpha_5 > \alpha_6)$.

*Let the set of interpretation be the cartesian product of the variable values. For sake of simplicity we have:* $\Phi_1 = (a, \alpha_1)$, $\Phi_2 = (b, \alpha_2)$, $\Phi_3 = (((\neg(a \wedge b) \wedge \neg(\neg a \wedge \neg b)) \vee c) \wedge (\neg(a \wedge \neg b) \wedge \neg(\neg a \wedge b))\neg c), \alpha_3)$, $\Phi_4 = ((\neg a \vee c) \wedge (a \neg c), \alpha_4)$, $\Phi_5 = ((\neg a \wedge b) \vee (a \wedge \neg b, \alpha_5)$, $\Phi_6 = ((\neg b \vee c) \wedge (b \vee \neg c), \alpha_6)$.

*Table 3.1 presents the possible alternative choices vectors.*

Table 3.1: Symbolic possibilistic vectors

| $\Omega$ | $\Phi_1$ | $\Phi_2$ | $\Phi_3$ | $\Phi_1$ | $\Phi_4$ | $\Phi_5$ | $\Phi_1$ | $\Phi_5$ | $\Phi_6$ |
|---|---|---|---|---|---|---|---|---|---|
| $abc$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $ab\neg c$ | 1 | 1 | $1-\alpha_3$ | 1 | $1-\alpha_4$ | 1 | 1 | 1 | $1-\alpha_6$ |
| $a\neg bc$ | 1 | $1-\alpha_2$ | $1-\alpha_3$ | 1 | 1 | $1-\alpha_5$ | 1 | $1-\alpha_5$ | $1-\alpha_6$ |
| $a\neg b\neg c$ | 1 | $1-\alpha_2$ | 1 | 1 | $1-\alpha_4$ | $1-\alpha_5$ | 1 | $1-\alpha_5$ | 1 |
| $\neg abc$ | $1-\alpha_1$ | 1 | $1-\alpha_3$ | $1-\alpha_1$ | $1-\alpha_4$ | $1-\alpha_5$ | $1-\alpha_1$ | $1-\alpha_5$ | 1 |
| $\neg ab\neg c$ | $1-\alpha_1$ | 1 | 1 | $1-\alpha_1$ | 1 | $1-\alpha_5$ | $1-\alpha_1$ | $1-\alpha_5$ | $1-\alpha_6$ |
| $\neg a\neg bc$ | $1-\alpha_1$ | $1-\alpha_2$ | 1 | $1-\alpha_1$ | $1-\alpha_4$ | 1 | $1-\alpha_1$ | 1 | $1-\alpha_6$ |
| $\neg a\neg b\neg c$ | $1-\alpha_1$ | $1-\alpha_2$ | $1-\alpha_3$ | $1-\alpha_1$ | 1 | 1 | $1-\alpha_1$ | 1 | 1 |

We notice that:

- In structure (1): We found that the Symmetric Pareto ordering and the Leximin ordering are totally able to capture the ordering of the CP-net
- In structure (2): Symmetric Pareto is unable to provide the CP-net ordering. In fact, it fails to compare the interpretations $a\neg b\neg c$ and $\neg a\neg b\neg c$, while, there exist a directed chain between those interpretations and we have $a\neg b\neg c \succ \neg a\neg b\neg c$ according to the CP-net ordering. Indeed the two interpretation vectors are not comparable due to the fact that $1 - \alpha_1 < min(1 - \alpha_4, 1 - \alpha_5)$ and $1 < max(1 - \alpha_4, 1 - \alpha_5)$. Otherwise, the Leximin order is able to capture this order exactly.

Table 3.2: Methods of preference representation summary

| | CP-NET | POSSIBISTIC LOGIC | POSSIBISTIC SYMBOLIC |
|---|---|---|---|
| LOGICAL REPRESENTATION | No | Set of prioritized goals | Set of prioritized goals + symbolic weights |
| PRINCIPLE | Ceteris Paribus | Minimal or maximal specificity | Symmetric Pareto or Leximin |
| DOMINANCE TESTING | Chain of worsening flips | Vector comparison | Vector comparison |
| ORDER | Partial | Total | Partial |
| PRIORITY | partially induced by the structure | Defined by decision maker | Defined by decision maker |
| GRAPH REPRESENTATION | Yes (DAG + CP-tables) | No | No |

- In structure (3): The Leximin ordering fails to provide the CP-net order while the Symmetric Pareto is fully able to capture it.

Therefore, the Symmetric Pareto is not able to compare two alternative choices where there exists a variable with more than one child (structure 2). But, in structure 3 we noticed that the Leximin ordering is more precise than the CP-net ordering, as it is able to compare two non comparable interpretations of the CP-net.

Although, that the symbolic possibilistic base overcomes the problem of the possibilistic base namely the order induced is more precise and significant, it still suffer from the absence of the graphical component.

## 3.6 Discussion

Although that many researches focused on reproducing the same order induced by the CP-net, it reveals until now that the additional implicit information used by the CP-net are difficult to determine or to encode. But, many proposed methods based on the possibility theory were provided, producing ordering that are somewhat rival to the CP-net ranking.

Table 3.2 gives an overview of the different methods, presenting their resemblances, differences and limitations .

We do not have precise criteria on which we can evaluate orders. But, each order is somewhat logic and coherent with a special case. Also, we can determine if incomparable interpretations are not wanted, because it may happen in some cases that two interpretations could not be compared. But one thing that is strange, the fact that the CP-net do not react in the same manner towards all the variables.

Despite that, one powerful feature in the CP-net is that it has a graphical component which displays the preference relations in a very obvious manner. While, The possibilistic framework for handling preferences relies on very strong basis where we can get profit from the logic counterpart of that framework. In the next chapter, we are going to propose a new representation model based on the possibility theory which overcomes the limitations of the both settings.

## 3.7 Conclusion

One of the major contribution of the possibilistic logic, besides its logical nature, is that the decision maker is able to define the relative importance between the variables. We noticed that, the order induced when using symbolic weights is more faithful to user expectancy. Firstly, it ensures the transitivity of priorities. Secondly, it reveals that it is somehow more expressive than the CP-net. In this chapter, we presented the characteristics of each method with highlighting their limitations.

In the next chapter, we will propose a new representation method based on the possibility theory. It takes advantage from of the graphical component of CP-net and it is inspired the symbolic possibilistic base as it uses symbolic weights.

Chapter 4

# Possibilistic networks for preferences

## 4.1 Introduction

As detailed in Chapters 1 and 2, existing methods of preference representation can be classified into two main categories graphical ones and non graphical ones. While the graphical methods suffer from a lot of limitations, non graphical methods suffer from the lack of an illustrative descriptive component.

In Chapter 3, we proposed a generic comparison and a study of different methods existing in the possibilistic framework and the CP-nets. We roughly presented the weaknesses of each approach. During this decade, there have been emergence of the representation of preference under the possibilistic framework. Although that this theory was perfectly able to deal with the preferences, it still until now suffer from the absence of the graphical component. One of the principle useful aspects of the CP-net is the existence of a graph which is able to express efficiently and effortlessly the dependence between the variables. In spite of the its defective ordering this approach still one of the well-known and mostly used method.

This chapter proposes a new possibilistic approach for handling preferences, offering the possibility to express preferences under a graphical structure. Moreover, it takes advantage of the logical side of the possibility theory. Besides, it is able to highlight the different dependencies between the variables.

The main sections of this chapter are: Section 4.2 introduces the structure of the representation model. Section 4.3 presents the two queries asked namely the ordering and the optimization queries. After that, Section 4.4 presents a comparison of our method with the CP-net. Finally Section 4.5 is dedicated to present the implementation of a toolbox able to perform the principle preference queries.

Principle results of this chapter are accepted to be published in (Ben Amor, Dubois, Gouider, & Prade, 2014a).

## 4.2 Modeling preferences with possibilistic networks

Possibilistic networks are decomposed into two main components. Namely, the graphical component and the numerical component. In this section we will propose a definition of these components in the preference representation setting.

To illustrate the idea of representing preferences by means of possibilistic network, we shall use the following running example inspired from the CP-net literature (Boutilier et al., 2004) (observe that $a$ and $d$ are not symmetric).

**Example 4.1.** *Let us consider a simple example about a night dressing with 4 variables standing for shirt (S), trousers (T), jacket (J) and shoes (H) s.t* $D(S) = \{black(s), red(\neg s)\}$, $D(T) = \{black(t), red(\neg t)\}$, $D(J) = \{red(j), white(\neg j)\}$ *and* $D(H) = \{white(h), black(\neg h)\}$.
*The preference conditional set is:*

| |
|---|
| The user prefers to wear a black shirt to a red one. |
| He prefers to wear black trousers to red ones. |
| If he wears a black shirt and black trousers, he prefers to wear a red jacket to a white one. |
| If he wears a black shirt and red trousers, he prefers to wear a white jacket. |
| If he wears a red shirt and black trousers, he prefers to wear a red jacket. |
| If he wears a red shirt and red trousers, he prefers to wear a white jacket. |
| If he wears a red jacket, he prefers to wear white shoes to black ones. |
| If he wears a white jacket, he prefers to wear black shoes. |

### 4.2.1 Graphical component

Our representation for preferences is graphical in nature, and exploits conditional preferential independence in structuring preferences of a user. The nature of the relations in this possibilistic network are not enough strong anymore. Since, they only translate conditional preference relations. Our aim in this representation is to provide a graph of the possibilistic framework to capture qualitative statements. Recall that CP-net rely on the Ceteris Paribus principal which is not the case here. As the preference description is assumed to be given under the form of conditional statements of the form $c : a \succ \neg a$ where $c$. And, stands for the specification of a context in terms of variables. Unconditional preferences correspond to the case where $c$ is the tautology $\top$. The graphical structure of the network is then directly determined from this description (as in the CP-net case). Therefore:

- A node: is a choice variable.
- An edge: a preferential independence.

The same characteristics of the possibilistic network are also applied here. The only change occurs in the meaning of the relations.

### 4.2.2 Possibilistic preference tables

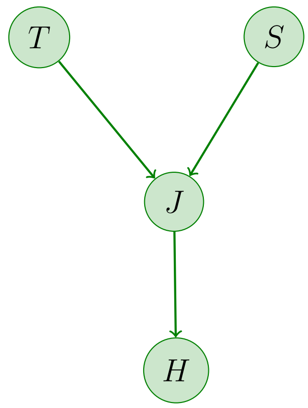
As in the basic possibilistic networks, we associate to each node a possibilistic preference table ($\Pi P$-table for short) (and thus to each variable) defined in the following way. To each preference of the form $c : a \succ \neg a$, pertaining to a variable $A$ whose domain is $\{a, \neg a\}$, is associated the conditional possibility distribution $\pi(a|c) = 1$ (because it is the preferred value) and $\pi(\neg a|c) = \alpha$ where $\alpha$ is a symbolic weight such that $\alpha < 1$. We write $\pi(\cdot|\top) = \pi(\cdot)$ for independent variables. Values are assigned in the following way:

- We assign a weight equal to 1 to the most preferred value, knowing the parents configuration. Therefore, each column will be normalized.
- We assign, to other values, a symbolic weight inferior to 1.

**Example 4.2.** *Figure 4.1 gives the possibilistic graph associated to the above example. For instance:*

- *For the variable $T$ we prefer $t$ to $\neg t$ because $\pi(t) = 1$ and $\pi(\neg t) = \alpha$ with $\alpha < 1$;*
- *The corresponding conditional possibility distribution of the variable $H$ is $\Pi(h|j) = 1$ and $\Pi(\neg h|j) = \epsilon_1$, $\Pi(\neg h|\neg j) = 1$ and $\Pi(h|\neg j) = \epsilon_2$ such that $max(\epsilon_1, \epsilon_2) < 1$. Therefore, when having $j$ we prefer $h$. But we can not decide whether $\neg j$ and $h$ are*

| $\Pi(t)$ | $\Pi(\neg t)$ |
|---|---|
| 1 | $\alpha$ |




| $\Pi(s)$ | $\Pi(\neg s)$ |
|---|---|
| 1 | $\beta$ |

| $\Pi(.\|.)$ | $ts$ | $t\neg s$ | $\neg ts$ | $\neg t\neg s$ |
|---|---|---|---|---|
| $j$ | 1 | $\delta_2$ | 1 | $\delta_4$ |
| $\neg j$ | $\delta_1$ | 1 | $\delta_3$ | 1 |

| $\Pi(.\|.)$ | $j$ | $\neg j$ |
|---|---|---|
| $h$ | 1 | $\epsilon_2$ |
| $\neg h$ | $\epsilon_1$ | 1 |

Figure 4.1: A possibilistic network

*preferred to $j\neg h$ since there is not a constraint between the two symbolic weights $\epsilon_1$ and $\epsilon_2$.*

## 4.3 Ordering

As recalled in Chapter 2, we have two types of possibilistic networks: product-based and min-based ones. As it was observed in Chapter 3 possibilistic bases retain only the worst satisfaction degree due to the use of the minimum function. Therefore, to avoid that problem we should use the product-based possibilistic networks.

Possibilistic networks orders are under the form of a possibility distribution. Thanks to conditional independence relations as exhibited by the graph, and using the product-based conditioning for increasing the discriminating power, we have in Example 4.2: $\pi(TSJH) = \Pi(H|J) * \Pi(J|TS) * \Pi(T) * \Pi(S)$.

We are then in position to compute the symbolic possibility degree expressing the satisfaction level of any interpretation. For instance, $\pi(\omega_4) = \Pi(\neg h|j) * \Pi(j|t\neg s) * \Pi(t) * \Pi(\neg s) = \epsilon_1\delta_2\beta$. Similarly, $\pi(\omega_3) = \Pi(h|j) * \Pi(j|t\neg s) * \Pi(t) * \Pi(\neg s) = \delta_2\beta$. Then, based on the fact that $\forall\ \alpha,\ \alpha < 1$, and $\forall\alpha,\ \beta,\ \alpha * \beta < \min(\alpha, \beta)$, we can define a *partial order* $\succ_\Pi$ between interpretations under the form of a possibility distribution.

One major virtue of this approach is the ability to handle the variable importance freely i.e you can define which are the most preferred configurations by setting constraints between the symbolic weights. In fact, given two interpretations $\omega_i$, $\omega_j \in \Omega$, $\omega_i \succ_{\Pi} \omega_j$ iff $\pi(\omega_i) > \pi(\omega_j)$. Thus, for instance, $\omega_3 \succ_{\Pi} \omega_4$. Besides, $\pi(\omega_6)= \delta_1$ and $\pi(\omega_{14}) = \alpha\delta_3$, thereby $\omega_6$ and $\omega_{14}$ remain incomparable.

However, if we further assume $\alpha < \delta_1$ expressing that the unconditional preference associated with a node $T$ is more important than the preference $ts : j \succ \neg j$, we become in position to establish that $\omega_6 \succ_{\Pi} \omega_{14}$. Therefore, the approach leaves the freedom of specifying the *relative importance* of preferences.

Assume that for each node, i.e. each variable $V_i \in V$, two *distinct* symbolic weights are used, one for the context where the preferences associated with *each* parent nodes are satisfied, one *smaller* for all the other contexts. For instance, the symbolic weights of the variable J become $\delta_1 > \delta_2 = \delta_3 = \delta_4$ and those of the variable H become $\epsilon_1 > \epsilon_2$. The partial order induced from the possibilistic network (without adding other constraints between symbolic weights) is then faithful to the inclusion order associated to the violated constraints. It is, in fact, exactly the same ordering. This is due to the non comparability between some symbolic weights (following from the use of product). Figure 4.2 shows the inclusion-based order induced by the possibilistic graph with these additional assumptions.

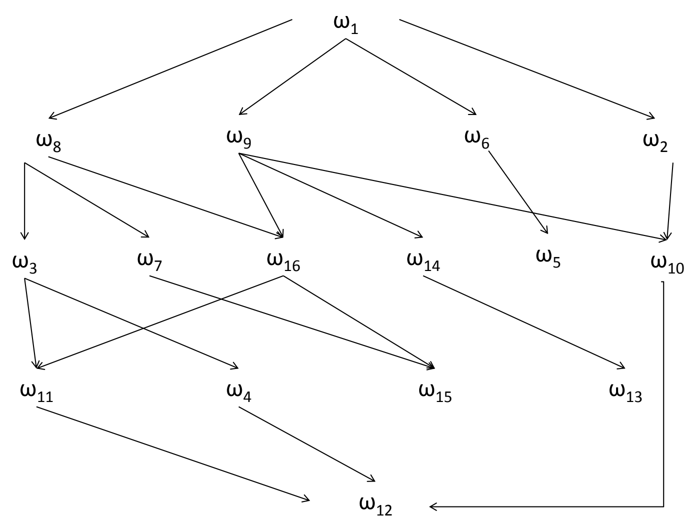



Figure 4.2: The inclusion-based ordering

It may be that CP-net orderings also respect the inclusion-based order found, although it has apparently never been investigated. Two main queries should be performed on preference representation models ordering queries and optimization queries. In the following section we are going to provide their corresponding algorithms. Those algorithms cover only the acyclic graphs.

### 4.3.1 Outcome optimization

We can easily determine the best interpretation among all the possible interpretations satisfied by the preference possibilistic network. Intuitively, to generate an optimal interpretation we simply need to sweep through the network from the top to the bottom just like in the CP-net. In each iteration we should set the value to its most preferred value (1) depending on the parents (if ever they exist). Indeed, despite that many interpretation can be judged equal, a unique best outcome is found.

Optimization queries can be answered using the following sweeping forward procedure (inspired from the CP-net), taking a linear time to the number of variables. This procedure exploits the considerable power of the graphical modeling of the preferential statements to easily find an optimal outcome:
It has two main steps:

- First, we choose the value of the independent variables. Its weight should be equal to 1.
- Second, for the dependent variables where all their immediate ascendent are assigned we choose the value equal to 1.

### 4.3.2 Dominance query

Finding an order of all the outcomes is easier than using the CP-net. We just have to compute the joint possibility distribution of all the outcomes. It is, in fact, a sweeping through procedure where in each time we compute gradually each corresponding possibility distribution. Then, after computing this, and revising the constraints between symbolic weights, we can deduce a partial order between them.

## 4.4 Comparison with CP-nets

While CP-nets are based on the Ceteris Paribus principle, possibilistic networks do not obey that latter principle as it can be seen on the previous example (where $\omega_6$ and $\omega_{14}$ are incomparable, while $\top : t \succ \neg t$). The order induced by the CP-net is a refinement of the possibilistic order $\succ_\Pi$, if no constraints about the relative importance of preferences are added.

CP-nets, in some sense, apply systematically the principle of Ceteris Paribus, as seen previously. However, in this approach, priorities associated to parents nodes can be determined and fixed by setting constraints over the symbolic weights. Therefore, we have the ability to decide about the variable importance. The basic ordering associated to a possibilistic network is just the inclusion-based ordering, which can then be completed by adding relative importance constraints. In particular, a complete ordering of the symbolic weights leads to a complete preordering of the interpretations, as exemplified now.

**Example 4.3.** *Figures 4.3 and 4.4 show, respectively, the order induced by the CP-net and possibilistic network of Example 1. Here we assume $\alpha = \beta < \delta_1 < \delta_2 = \delta_3 = \delta_4 < \epsilon_1 < \epsilon_2$. For instance, let us consider the interpretations $\omega_7$ and $\omega_{16}$. In contrast to the possibilistic network, which gives a total preorder, the CP-net considers these two interpretations as incomparable. We notice that both interpretations violate two preferences: associated to a parent and to a grandchild for $\omega_7$, and to two parents preferences for $\omega_{16}$. As expected, $\omega_7$ is preferred to $\omega_{16}$ in the possibilistic network as their possibility degrees are respectively $\pi(\omega_7) = \beta\epsilon_2$ and $\pi(\omega_{16}) = \alpha\beta$.*

Moreover, CP-nets are sometimes unable to represent some user preferences. This is illustrated by the following example.

**Example 4.4.** *Let us consider the following preferences ordering: $ab \succ \neg a\neg b \succ a\neg b \succ \neg ab$. For instance, $a$ stands for "vacations" and $b$ stands for "good weather". We observe that this complete preorder cannot be represented by a CP-net, while the possibilistic network can display it. Indeed, such preferences can be represented by a joint possibility distribution such that: $\pi(ab) > \pi(\neg a\neg b) > \pi(a\neg b) > \pi(\neg ab)$. Since any joint possibility distribution can be decomposed into conditional possibility distributions, any complete preorder can be represented by a possibilistic net. Here, we have $\top : a \succ \neg a$, $a : b \succ \neg b$ and $\neg a : \neg b \succ b$. It corresponds to the initial network with $\Pi P$-tables given in Table 4.1. which yields $\pi(ab) = 1 > \pi(\neg a\neg b) = \alpha > \pi(a\neg b) = \beta > \pi(\neg ab) = \alpha\gamma$ taking $\alpha > \beta$ and $\beta = \gamma$.*

Lastly, it is important to mention that one of the advantages of the possibilistic graph is

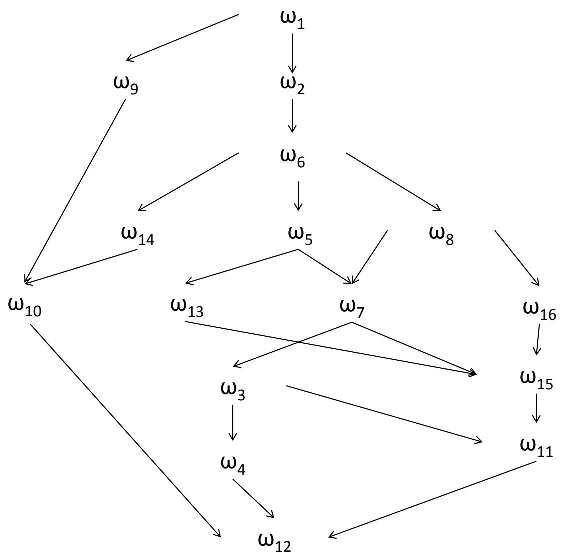



Figure 4.3: The order induced by the CP-net

Table 4.1: Preference conditional tables

| $\pi(a)$ | $\pi(\neg a)$ |
|---|---|
| 1 | $\alpha$ |

| $\pi(\cdot\|\cdot)$ | $a$ | $\neg a$ |
|---|---|---|
| $b$ | 1 | $\gamma$ |
| $\neg b$ | $\beta$ | 1 |

its ability to be translated into a possibility logic base (Benferhat et al., 2002) that can be used for inference. This bridges the approach presented here with the *direct* representation of preferences by a possibilistic logic base (Dubois et al., 2013b; Kaci & Prade, 2008).

## 4.5 Implementation

In this section we present the implementation of our method. The main purpose of our implementation is the construction of the ordering induced, and then the evaluation of these rankings. For this proposal, we have developed two main programs of two principal tasks. Namely, optimization and ordering.

In fact, given a possibilistic network for preference there are two principal procedures that should be implemented:

- Optimization : finding the best outcome;

| |
|---|
| $\omega_1$ |
| $\omega_2$ |
| $\omega_6$ |
| $\omega_5, \omega_8, \omega_9$ |
| $\omega_7, \omega_{10}$ |
| $\omega_3, \omega_{14}$ |
| $\omega_4, \omega_{13}, \omega_{16}$ |
| $\omega_{15}$ |
| $\omega_{11}$ |
| $\omega_{12}$ |

Figure 4.4: The order induced by the possibilistic network

- Ordering: rank-order all the interpretations.

## 4.6 Conclusion

This chapter has outlined a preliminary presentation of possibilistic networks as providing a convenient setting for preference representation. This setting remains close to the spirit of Bayesian nets, but is flexible enough, thanks to the introduction of symbolic weights, for capturing any ordering agreeing with the inclusion-based ordering.

# Conclusion

In this work, we provide a comparative study with respect to CP-nets, and previous attempts at a possibilistic modeling. We highlight the different limitations of the presented methods. Besides, we studied their corresponding orderings and their differences.

We found that despite their popularity CP-nets suffer from unavoidable problems. First, systematic application of priority over the variables. Second, the lack of that priority transitivity. Besides, we observed that, while the possibilistic framework overcomes these drawback, it still suffer from the absence of a graphical display.

Therefore, we have noticed that there are two main kinds of limitations. First, some methods lack the graphical component that allow to assess preference compactly. Second, remarkable problem on the ordering induced by some methods. Obviously, we wanted to propose a new model that is able to overcome these problems.

Moreover, another main contribution of this work is the proposal of a new approach, based on product-based possibilistic networks, for representing preferences. In this approach, possibility degrees may remain symbolic but stands for numbers. As we saw, the representation is particularly faithful to the user's preferences. The ordering between interpretations that can be obtained from this compact representation fully agrees with the inclusion ordering associated with the violation of preference statements. Besides, the relative importance of preferences can be easily taken into account when available.

Furthermore, we observed that the expressive power of our approach is even more important than the CP-nets. This latter, is sometimes unable to represent some kinds of preferences while the possibilistic network for preferences is perfectly able to display them.

Finally, further research is still needed for investigating the potential of possibilistic networks in greater detail. Since preference possibilistic networks deal only with acyclic graph, we aim at extending the application of this approach to be able to handle cyclic preference. Moreover, we can extend our approach in order to handle the uncertainty in both variable dependence and preference relation under the possibilistic framework. Besides, possibilistic networks can be studied in depth where we can compare the complexity of the queries performed to the other approaches. In addition to that, it would be interesting to apply our approach in several real problems.

# References


Ben Amor, N., & Benferhat, S. (2005). Graphoid properties of qualitative possibilistic independence relations. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Sys.*, *13*(1), 59-96.

Ben Amor, N., Benferhat, S., & Mellouli, K. (2003). Anytime propagation algorithm for min-based possibilistic graphs. *Soft Computing*, *8*(2), 150-161.

Ben Amor, N., Dubois, D., Gouider, H., & Prade, H. (2014a). Possibilistic networks: A new setting for modeling preferences. *Scalable Uncertainty Management, eighth International Conference, SUM 2014, , September 15-17, 2014. Proceedings*.

Ben Amor, N., Dubois, D., Gouider, H., & Prade, H. (2014b). Preference representation under the possibilistic framework : A comparative study. *Logique floue et ses applications, October 23-27*.

Benferhat, S., Dubois, D., Garcia, L., & Prade, H. (2002). On the transformation between possibilistic logic bases and possibilistic causal networks. *Int. J. of Approximate Reasoning*, *29*(2), 135–173.

Benferhat, S., Dubois, D., Kaci, S., & Prade, H. (2001). Bridging logical, comparative, and graphical possibilistic representation frameworks. In (Vol. 44, p. 422-431).

Benferhat, S., Dubois, D., & Prade, H. (1992). Representing default rules in possibilistic logic. In *Kr* (p. 673-684). Morgan Kaufmann.

Benferhat, S., Dubois, D., & Prade, H. (1999). Towards a possibilistic logic handling of preferences. *Appl. Intell. IJCAI*, *14*, 303-317.

Benferhat, S., & Garcia, L. (1997). Dealing with locally-prioritized inconsistent knowledge bases and its application to default reasoning. *Applications of Uncertainty Formalisms*.

Benferhat, S., & Smaoui, S. (2007). Hybrid possibilistic networks. *Int. J. Approx. Reasoning*, *44*(3), 224–243.

Bienvenu, M., Lang, J., & Wilson, N. (2010). From preference logics to preference languages, and back. In F. Lin, U. Sattler, & M. Truszczynski (Eds.), *Proc. 12th int. conf. on principles of knowledge representation and reasoning (kr 2010), toronto, may 9-13.*

Boutilier, C., Brafman, R., Domshlak, C., Hoos, H., & Poole, D. (2004). Cp-nets: A tool for representing and reasoning with conditional *ceteris paribus* preference statements. *J. Artif. Intell. Res. (JAIR)*, *21*, 135–191.

Boutilier, C., Brafman, R., Hoos, H., & Poole, D. (1999). *Reasoning with Conditional Ceteris Paribus Preference Statements.* Morgan Kaufmann Publishers.

Brafman, R., & Domshlak, C. (2002). Introducing variable importance tradeoffs into CP-nets. In A. Darwiche & N. Friedman (Eds.), *Proc. 18th conf. in uncertainty in artificial intelligence (uai '02), edmonton, alberta, aug. 1-4* (p. 69-76).

Cornelio, C., Goldsmith, J., Mattei, N., Rossi, F., & Venable, K. (2013). Dynamic probabilistic cp-nets. In *Multidisciplinary workshop on advances in preference handling* (p. 7).

Damien, B., Zanuttini, B., Fargier, H., & Mengin, J. (2013). Probabilistic conditional preference networks. *CoRR*, *abs/1309.6817*.

de Amo, S., Bueno, M., Alves, G., & da Silva, N. F. F. (2012). Cprefminer: An algorithm for mining user contextual preferences based on bayesian networks. In *Ictai* (p. 114-121).

Domshlak, C., Hüllermeier, E., Kaci, S., & Prade, H. (2011). Preferences in AI: An overview. *Artif. Intell.*, *175*(7-8), 1037-1052.

Dubois, D., Kaci, S., & Prade, H. (2005). Expressing preferences from generic rules and examples - a possibilistic approach without aggregation function. In *Ecsqaru* (Vol. 3571, p. 293-304).

Dubois, D., Kaci, S., & Prade, H. (2006). Approximation of conditional preferences networks "CP-nets" in possibilistic logic. In *Proc. 15th Int. Conf. on Fuzzy Systems (FUZZ-IEEE), Vancouver, July 16-21.*

Dubois, D., Lang, J., & Prade, H. (1994). Possibilistic logic. In D. Gabbay, C. Hogger, J. Robinson, & D. Nute (Eds.), *Handbook of Logic in Artificial Intelligence and Logic Programming, Vol. 3* (p. 439-513). Oxford University Press.

Dubois, D., & Prade, H. (1988). *Possibility theory: An approach to computerized processing of uncertainty.* Plenum Press.

Dubois, D., & Prade, H. (2008). Qualitative possibility theory in information processing. In *Forging new frontiers: Fuzzy pioneers ii* (pp. 53–83). Springer.

Dubois, D., Prade, H., & Touazi, F. (2013a). Conditional preference nets and possibilistic logic. In *Ecsqaru.*

Dubois, D., Prade, H., & Touazi, F. (2013b). Conditional Preference-nets, possibilistic logic, and the transitivity of priorities. In M. Bramer & M. Petridis (Eds.), *Research and development in intelligent systems xxx, proc. 33rd sgai int. conf. on innovative techniques and applications of artificial intelligence (ai'2013), cambridge, uk, dec. 10-12* (p. 175-184).

Dubois, D., Prade, H., & Touazi, F. (2013c). Réseaux de préférences conditionnelles et logique possibiliste. *Septièmes Journées d'Intelligence Artificielle Fondamentale*, 127.

et al., J. G. (2008). The computational complexity of dominance and consistency in cp-nets. *J. Artif. Intell. Res. (JAIR)*, *33*, 403-432.

Gérard, R., Kaci, S., Prade, H., et al. (2007). Ranking alternatives on the basis of generic constraints and examples-a possibilistic approach. In *Ijcai* (p. 393-398).

Glenn Shafer, G. (1976). *A mathematical theory of evidence.* Princeton University Press.

HadjAli, A., Kaci, S., & Prade, H. (2008). Database preferences queries - a possibilistic logic approach with symbolic priorities. In S. Hartmann & G. Kern-Isberner (Eds.), *Foiks* (Vol. 4932, p. 291-310). Springer.

Kaci, S., Dubois, D., & Prade, H. (2004). Representing preferences in the possibilistic setting. In *Preferences.* IBFI, Schloss Dagstuhl, Germany.

Kaci, S., & Prade, H. (2008). Mastering the processing of preferences by using symbolic priorities in possibilistic logic. In M. Ghallab, C. D. Spyropoulos, N. Fakotakis, & N. M. Avouris (Eds.), *Proc. 18th europ. conf. on artificial intelligence (ecai'08), patras, july 21-25* (p. 376-380). IOS Press.

Kaci, S., & van der Torre, L. (2008). Reasoning with various kinds of preferences: logic, non-monotonicity, and algorithms. *Annals of Operations Research*, *163*(1), 89-114.

Kaci, S., & van der Torre, L. (2008). Reasoning with various kinds of preferences: logic, non-monotonicity, and algorithms. *Annals OR*, 89-114.

Pearl, J. (1985). Bayesian networks: A model of self-activated memory for evidential reasoning. In *Proc. of cognitive science society (css-7).*

Wilson, N. (2011). Computational techniques for a simple theory of conditional preferences. *Artif. Intell.*, *175*(7-8), 1053-1091.

Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, *1*, 3–28.