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Evaluation of possibilistic influence diagrams using junction trees

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Summary

Influence diagrams are well-known decision frameworks in the literature. They present an alliance between decision theory, graph theory and probability theory, in order to represent decision problems and define their optimal strategy through evaluation algorithms.

Many researchers have focused on this problem, which is known as NP-hard, by proposing several evaluation methods that can be classified into two main categories: (i) direct evaluation (operating directly on the diagram) or (ii) indirect evaluation which is based on the transformation of the initial diagram into a secondary structure such as decision trees, Bayesian networks, junction trees etc.

The majority of these works refer to probability theory, but in light of possibility theory evolution, possibilistic influence diagrams have been proposed as a counterpart of classical influence diagrams, within this theory. Hence, the need to develop new evaluation algorithms specific to these diagrams.

Therefore, the aim of this work is to propose a possibilistic adaptation of Jensen's evaluation method which is based on the transformation of influence diagrams into junction trees. The choice of this secondary structure is justified by the fact that it allows evaluation of huge influence diagrams contrary to the procedure based on transformation into decision trees. It provides also, reduction in complexity compared to the procedure of transformation into Bayesian networks, since these models must in turn be transformed into junction trees when the network contains cycles.

Résumé

Les diagrammes d'influence sont des modèles graphiques décisionnels très répondus dans la littérature. Ils présentent une alliance entre la théorie de décision, la théorie des graphes et la théorie des probabilités, afin de représenter des problèmes décisionnels et définir leur stratégie optimale à travers des algorithmes d'évaluation.

Beaucoup de chercheurs ont mis l'accent sur ce problème, qui est connu comme un problème NP-difficile, en proposant plusieurs méthodes d'évaluation qui peuvent être classées en deux catégories principales: (i) évaluation directe (opérant directement sur le diagramme) ou (ii) évaluation indirecte qui se base sur la transformation du diagramme initial en une structure secondaire comme les arbres de décision, les réseaux Bayésiens, les arbres de jonctions etc.

La majorité de ces travaux se réfèrent à la théorie des probabilités, mais avec l'évolution de la théorie des possibilités, les diagrammes d'influence possibilistes ont été proposés comme une contrepartie des diagrammes d'influence standards dans le cadre de cette théorie. D'où la nécessité de développer de nouveaux algorithmes dévaluation propres à ces diagrammes.

Donc, le but de ce travail est de proposer une adaptation possibiliste de la méthode d'évaluation de Jensen qui se base sur la transformation des diagrammes d'influence en arbres de jonction. Le choix de cette structure secondaire, se justifie par le fait qu'elle permet, d'évaluer des diagrammes de grande taille contrairement à la procédure se basant sur la transformation en arbres de décision. Elle procure, également, la réduction de la complexité par rapport à la procédure de transformation en réseaux Bayésiens, puisque ces derniers doivent à leur tour être transformés en arbres de jonction quand le réseau contient des boucles.

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General introduction

Decision theory or decision analysis is a discipline that has emerged to support decision making under uncertainty. It allows a formal representation of decision problems as well as determination of the optimal strategy that meets intended goals. Yet, in order to provide an optimal strategy relative to a decision problem we should clearly define the context of decisions to be taken and the environment to which they are associated.

Indeed, graphical decision models allow a clear description of the outlined problem with a simple representation of different decision scenarios and an ease computation of the value of each one. In the literature, we can specify three main families of decision models which are: decision trees [29], influence diagrams [1] and valuation based systems [31].

Within existing models, we are in particular interested to influence diagrams ($\mathcal{ID}$s), where the basics of these graphical representations were introduced for the first time by Miller et al in 1976 as a "front end" framework and then adapted to decision analysis by Howard and Matheson in 1984 [27, 30].

Since then, there have been several researches carried out to deal with influence diagrams and their evaluation which remains until now known as NP-hard problem [28]. Therefore, the majority of these approaches were proposed in a probabilistic context [1, 3, 4, 13, 30].

However, in real world it is not always obvious to provide probability distributions representing uncertainty relative to the problem's variables. Indeed, it's easier to express them qualitatively by defining a rank between the different states of a given variable. Besides, it's more leisurely to provide a preference order towards different strategies than estimating numerical values. This promotes the use of possibility theory which is the context of our work.

Nevertheless, in this work we deal with possibilistic influence diagrams that

can be defined as an extension of classical influence diagrams with possibilistic quantification. More precisely, the aim of this work is to present a new approach to evaluate these models using strong junction trees. This proposal consists on a possibilistic adaptation of Jensen's method presented in [13].

This report is organized as follows:

In chapter 1, we start by presenting classical influence diagrams and the principal methods to evaluate them which can be classified into direct or indirect evaluation methods. Then, in chapter 2, we define possibilistic influence diagrams and different variants of these models, as well as existing methods to evaluate them. Finally, in chapter 3 we focus on our proposal which consists on a new method to evaluate possibilistic influence diagrams for both cases (i.e. quantitative or qualitative). Then, we provide a brief comparison between our approach and existing methods, in addition to the global architecture of our experimentation.

Chapter 1

Introduction to influence diagrams

1.1 Introduction

In order to solve any decision problem and determine its optimal solution, we have to go first through the representation of this problem. Indeed, a formal modeling of complex problems offers a simple and clear description of the different variables involved in this problem and different relationships between them.

In the literature, we can distinguish several graphical decision models which are proposed to bridge the gap between knowledge formulation and decision analysis [30]. In this chapter, we will focus on one of these graphical models which are influence diagrams. These frameworks are known as powerful tools for representing and solving decision problems under uncertainty.

Mainly this chapter is composed of two sections: section 1.2 presents the definition of classical influence diagrams and section 1.3 proposes the most known methods available for evaluating these models.

1.2 Definition of influence diagrams

Influence diagrams ($\mathcal{ID}$s) offer a compact and natural modeling of decision problems under uncertainty. They can be defined as an alternative to decision trees as well as an extension to Bayesian networks. Thus, these graphical models overcome the limits of decision trees to represent decision problems and inherit the main features of Bayesian networks.

Formally, $\mathcal{ID}$s can be defined as an assembly of graphical and numerical components. The first one, outlines the different variables of problems and emphasizes the dependency/independency relationships between them. While the second component, resumes a range of various values describing the uncertainty degrees relative to these variables.

1.2.1 Graphical component

The graphical component is composed by a set of nodes and arcs which connect these nodes and describe the dependencies between them. Similar to Bayesian networks, the structure of influence diagrams is a directed acyclic graph ($\mathcal{DAG}$) denoted by $\mathcal{G}$ =(N,A), while N represents the nodes and A represents the arcs.

The set of nodes N offers a description of different elements of the system or problem that we aim to study. N includes three subsets C, D and V defined as follows:

- ***Chance nodes*** $C_i \in C = \{C_1, ..., C_n\}$, depict a set of random uncertain variables representing states of the world. The values of these variables are affected by each others according to the relations of dependency between them, but not affected by decisions to be made. They are usually drawn as circles or ovals.
- ***Decision nodes*** $D_i \in D = \{D_1, ..., D_p\}$, depict a set of decisions under control of the decision maker. Schematically, they are represented by squares, according to their temporal order.
- ***Value nodes*** $V_i \in V = \{V_1, ..., V_q\}$, depict a set of result variables which represent the objective sought to maximize. They measure the expected utility of the problem. And, their values are quantified by the combination of their parents. They are drawn as diamonds.

The set of arcs A exhibits dependencies and relationships between all the nodes. According to the type of nodes that connects, an arc can be:

- ***Conditional arc***: Connecting two nodes with a chance node or value node as a target.

- ***Informational arc***: Connecting two nodes with decision node as a target.

Similar to Bayesian networks, all these nodes and arcs describe the topology of the graph. This topology covers three types of configurations for any connected nodes A and B with an intermediate node C (figure 1.1).

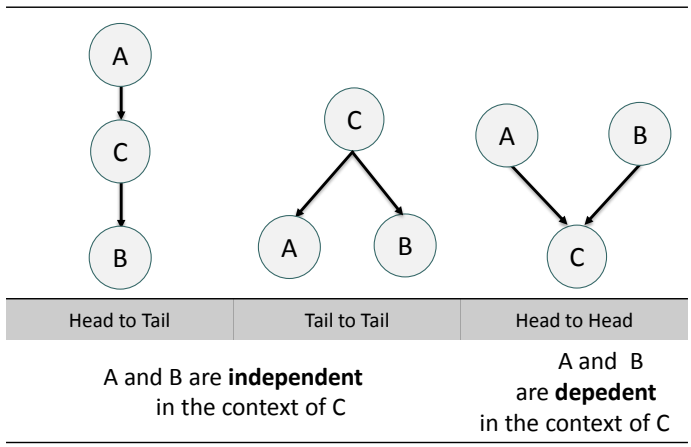

Figure 1.1: Conditional independency

Example 1.1. *Let us consider the figure 1.2 to represent the graphical component of an influence diagram. The outlined structure is defined by two chance nodes* (A_1, A_2)*, two decision nodes* (D_1, D_2) *and a value node* V*.*

1.2.2 Numerical component

The numerical component evaluates the dependencies of the graph. It quantifies the conditional arcs and allows the computation of the optimal solution. We attribute for each chance node C_i a conditional probability distribution. This distribution defines the different values assigned to each instance c_{ij} of C_i, and it's specified depending on $Par(C_i)$. Where $Par(C_i)$ is the set of parents of C_i in the graph and it may contains chance and decision nodes.

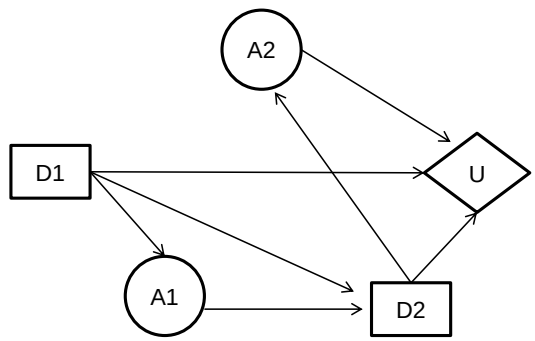

Figure 1.2: Example of graphical component of an influence diagram

The conditional probability for each chance node should be normalized such as:

$$\sum_{c_{ij} \in \Omega_{C_i}} P(c_{ij}|Par(C_i)) = 1 \tag{1.1}$$

where c_{ij} is an instance of C_i and Ω_{C_i} the domain of C_i.

If $Par(C_i) = \emptyset$, the equation 1.1 defines an a priori probability which can be written as follows:

$$\sum_{c_{ij} \in \Omega_{C_i}} P(c_{ij}) = 1 \tag{1.2}$$

Semantically, these probabilities exhibit an uncertainty degree for a given state. Once new evidence is observed, the probability degree associated to each chance node in the graph should be updated. The updating process depends on the d-separation criterion for IDs which can be defined based on the type of connection between variables (figure 1.1) such as:

Definition 1.1. *Given two variables A and B $\in$ N in a $\mathcal{DAG}$ G = (N,A), connected with an intermediate node C. We can say that A and B are d-separated if:*
- The connection is tail to head or tail to tail and C is instantiated.
- The connection is head to head and neither C nor any of its descendants is instantiated.

According to Jensen [20], this definition is adapted for influence diagrams with ignorance of utility nodes and informational arcs, which allows the definition of the following proposals:

Proposition 1.1. *Let D_j a decision node in an ID and C_i a chance node $\in Par(D_j)$ where $i < j$ then:*

- *(i) C_i and D_j are d-separated: $P(C_i|D_j) = P(C_i)$.*

- *(ii) Let W be any set of variables prior to D_j in the temporal ordering. Then C_i and D_j are d-separated given W: $P(C_i|D_j, W) = P(C_i|W)$.*

As mentioned above, the probabilities are associated only with chance nodes. The decision nodes represent a set of alternatives under control of decision maker. The choice of one or another alternative for every decision leads to several consequences (strategies) quantified by utility functions denoted U. These utilities constitute the quantification relative to value nodes in the context of $\mathrm{Par}(V_i)$.

For any $\mathcal{ID}$, since decision nodes are not instantiated, the chain rule relative to the diagram can be defined for any decision policy such as:

$$P(C|D) = \prod_{C_i \in C} P(C_i|Pa(C_i)). \tag{1.3}$$

Example 1.2. *In this example we present the numerical component relative to influence diagram presented in figure 1.2. The tables 1.1 and 1.2 present the conditional probabilities assigned respectively to chance nodes A_1 and A_2, and the table 1.3 presents the utilities assigned to value node V. The probabilities or utilities are assigned in context of the parents of each node.*

A_1	D_1	$P(A_1\|D_1)$
T	T	0.6
T	F	1
F	T	0.4
F	F	0

Table 1.1: Conditional probability of A_1

A_2	D_2	$P(A_2\|D_2)$
T	T	0.5
T	F	0.2
F	T	0.5
F	F	0.8

Table 1.2: Conditional probability of A_2

D_1	D_2	A_2	$u(D_1, D_2, A_2)$
T	T	T	10
T	T	F	1
T	F	T	5
T	F	F	5
F	T	T	2
F	T	F	7
F	F	T	0
F	F	F	0

Table 1.3: Utility function of (D_1, D_2, A_2)

1.3 Evaluation of influence diagrams

The concept of utility is a very old concept that has emerged since the classical economics. Utility is described as a level of satisfaction associated to an event or decision in order to evaluate it. These utilities can be ordinal or cardinal. Cardinal utility can be defined when satisfaction degrees are measurable and quantitative. While, ordinal utility describes an order of preferences among different alternatives.

In a probabilistic framework, decision analysis is essentially based on the notion of expected utility (EU). This concept was proposed for the first time by Von Neumann and Morgenstern in 1944 [26]. The principle of the expected theory is applicable to the present to solve decision problems, in particular to evaluate influence diagrams, which is the subject of this section.

Evaluating influence diagrams is the process adopted to determinate the optimal solution that solve an outlined decision problem. It refers to the inference task applied for the diagram by choosing the best decision alternatives in the aim to maximize the utility function and to obtain the optimal strategy. An optimal strategy can be defined as follows:

Definition 1.2. *The optimal strategy is a sequence of decision rules or policies denoted by $\Delta = \{\delta_i, ..., \delta_p\}$ where δ_i is the best decision rule relative to D_i, providing the highest utility, with regards to a decision criterion O.*

The determination of this strategy which amounts to evaluate influence diagrams has a theoretical complexity equivalent to the complexity of inference in Bayesian networks which is NP-hard. To solve this problem, we can distinguish in the literature several methods which can be summarized into two broad categories:

direct evaluation and indirect evaluation.

1.3.1 Properties of influence diagrams

Shachter [30] mentioned that to be evaluated, an influence diagram should be ***oriented*** (containing a value node), ***proper*** (with unambiguous representation) and ***regular***.

Definition 1.3. *We can say that an influence diagram is* ***regular*** *if its $\mathcal{DAG}$ satisfies the following properties:*

- *There is no cycle.*
- *The value node doesn't have children.*
- *"No forgetting": There is a directed path comprising all decision nodes.*

By respect to the "No forgetting" property, all decision nodes of any regular influence diagram should be ordered, then we can define the notion of *decision windows* such as:
For each decision D_i in $\{D_1...,D_p\}$ a set of ordered decisions, there exists a set of chance variables $SC_i \in \Omega = \{SC_i...,SC_p\}$ observed in the moment of taking decision D_i. This set of chance nodes is called decision window of D_i.

1.3.2 Direct evaluation of influence diagrams

This method consists on computing of the optimal strategy while performing direct manipulations on the structure of the diagram. These transformations should preserve the semantic of the outlined problem. The principles of direct evaluation was proposed by Shachter in 1986 [30], and they consist on performing the algorithm 1.1.

This algorithm presents four operations performed to simplify the structure of the diagram [20, 30] which are:

Proposition 1.2. ***Barren node removal****: A* ***barren node*** *is a node without children or all its children are barren. It can be a chance node or decision node. These nodes have no effect on the calculation of the optimal strategy, so they can be easily removed.*

Algorithm 1.1: Direct evaluation

```
Data: In: An influence diagram
Result: The optimal strategy of the diagram and its value.
begin
    - Make sure that the influence diagram is regular.
    - Eliminate barren nodes.
    - Let Suc_C_i be a set of nodes containing successors of chance node C_i.
    while Par(V_i) ≠ ∅ do
        if ∃ chance node C_i ∈ Par(V_i) then
            Remove C_i
        else
            if ∃ decision node D_i ∈ Par(V_i) then
                Remove D_i and barren nodes.
            else
                Find a chance node C_i where Suc_C_i ∩ D = ∅
                while Suc_C_i ≠ ∅ do
                    if ∃ C_j where (C_i) ∈ Par(C_j) then
                        Reverse arc between C_i and C_j
                Remove C_i
end
```

Theorem 1.1. ***Chance node removal****: If a chance node C_i is parent of only a value node V_i or a set of value nodes $\{V_i, .., V_k\}$, it can be removed from the diagram and the value node inherits the conditional parents of C_i. In the case where C_i is parent of a set of value nodes, these nodes are also removed and substituted by a single value node. After this operation we have:*

$$U^* = \sum_{C_i} P(C_i|Par(C_i))[\sum_{i=1}^{k} U_i] \tag{1.4}$$

where $\sum_{C_i}$ denotes the operation of marginalization over C_i.

Theorem 1.2. ***Decision node removal****: If a decision node D_i is parent of only a value node V_i or a set of value nodes $\{V_i, .., V_k\}$,and all other parents of value node(s) are parents of D_i. This node besides the value node(s) can be removed from the diagram. So, the optimal policy of D_i and potential of value node(s) will be respectively:*

$$\delta_i^* = \arg\max_{D_i} (\sum_{i=1}^{k} U_i) \tag{1.5}$$

$$U^* = \max_{D_i} (\sum_{i=1}^{k} U_i) \tag{1.6}$$

Theorem 1.3. ***Arc reversal****: If none of the removal operations can be performed, and exists two chance nodes C_i and C_j in the diagram connected with just one direct path (i,j). This arc can be reversed and nodes inherit each others' conditional parents by applying the Bayes' rule. After this operation, one of these nodes can be removed.*

These operations relative to direct evaluation are cumbersome and complex which has promoted the development of indirect methods.

1.3.3 Indirect evaluation of influence diagrams

The indirect evaluation of influence diagrams consists on mapping the diagram into a secondary structure and then applying the inference algorithm appropriate to this new model. In the literature, an influence diagram can be transformed into four auxiliary structures such as: decision trees, Bayesian networks, junction trees, and decision circuits. In the following, we will detailed the transformation into the first three intermediate structures.

1.3.3.1 Transformation of influence diagrams into decision trees

Decision trees ($\mathcal{DT}$s) are visual and analytical decision tools introduced for the first time by Raifa in 1968 [29]. They outline decision problems where each branch represents a possible decision scenario. Variables in $\mathcal{DT}$s are organized in the order of their observation. The root is the first observed variable and generally it's a decision variable.

These models are composed of chance nodes, decision nodes and leaves or endpoints which are extremities of branches. Arcs in decision trees have labels: Arcs incident from chance nodes are labeled with states of these nodes, and arcs incident from decision nodes are labeled with alternatives relative to these nodes. The quantification of leaves expresses utilities.

Evaluating decision trees means the generation of the optimal strategy i.e. the one that maximizes the expected utility criterion (MEU) expressed by:

$$MEU(X) = \arg\max \sum_{x_i \in X} P(x_i) * U(x_i) \tag{1.7}$$

The computation of the optimal strategy in decision trees is performed recursively from leaves to the root using the backwards reasoning procedure called *dynamic programming* [14].

Howard and Matheson [1] presented the first solution to evaluate influence diagrams using decision trees. Their method profits from the theoretical basis of decision trees and adopt them to evaluate influence diagrams. In the following a brief description of this procedure.

To be transformed into a decision tree, the influence diagram should satisfy the condition of "no forgetting", and its decision windows should be clearly defined (section 1.3.1). So, this ensures that the obtained decision tree preserves the same semantics of the initial diagram.

The principles of the method detailed in [1] can be defined as follows:

- Construction of *decision network* by adding arcs to the initial diagram, where each decision node D_i and its direct predecessors are directly connected to decision node D_{i+1}.
- Construction of *decision tree network* by reversing each arc that connects two chance nodes from different decision windows.
- Construction of the *decision tree* by reordering chance nodes.
- Quantification of the obtained decision tree by assigning conditional probabilities to chance nodes and utilities to the leafs. This quantification should be performed according to the numerical component of initial influence diagram.

After obtaining decision tree, the optimal strategy can be defined by evaluating this model through dynamic programming.

Example 1.3. *Figure 1.3 presents the decision tree corresponding to the transformation of the influence diagram presented in figure 1.2. The quantification of this decision tree resume the quantification of the initial diagram presented in tables: 1.1, 1.2 and 1.3. Using the maximal expected utility (MEU) and dynamic programming, the generated optimal strategy is* $\delta^* = \{(D_2 = T), (D_1 = T)\} = 5{,}5$.

1.3.3.2 Transformation of influence diagrams into Bayesian networks

Bayesian networks introduced by Pearl in 1985 [21] correspond to probabilistic graphical models used increasingly in widespread fields. They encode relationships

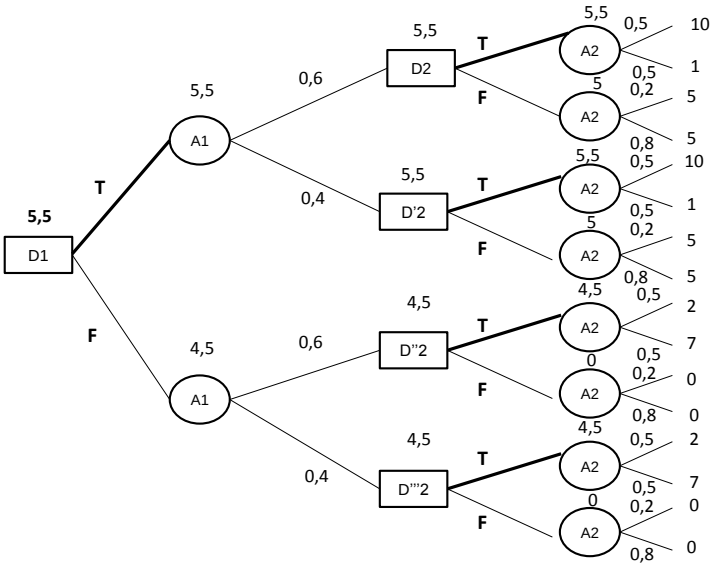

Figure 1.3: The decision tree corresponding to the influence diagram in figure 1.2 and its optimal strategy using MEU criterion.

among variables characterizing a given domain. Also, they present a reasoning model that offers the possibility to infer missing knowledge from a set of observed variables.

Bayesian networks, like other graphical models, are composed of graphical and numerical components and can be defined as an acyclic directed graph ($\mathcal{DAG}$) denoted by $\mathcal{G}$=(V, E). Where V is a set of nodes describing states of the world and E is the set of arcs connecting these nodes. The numerical quantification associated to this $\mathcal{DAG}$ expresses uncertainty by an a priori and conditional probabilistic distributions depending on the relationship between variables.

Inference or propagation in Bayesian networks was the subject of several researches [6, 22, 24]. This task depicts the impact of each variable on the remaining ones using probability calculations. Therefore, it involves updating of conditional probability of the graph after observing new information.

The described procedure was adopted by Cooper [4] in 1988, to introduce for the first time the evaluation of influence diagrams using Bayesian networks. This method has undergone several improvements [5], but the key idea remains the same. In this work, we will concentrate on the principles of Cooper's algorithm.

Since Bayesian networks are composed only of chance nodes, the transformation of influence diagrams using Bayesian networks consists on converting the decision nodes and value nodes of influence diagrams into chance nodes. So, chance nodes remain the same while decision and value nodes are transformed as follows:

The manipulation of each decision node consists on removing informational arcs into this node and assigning an equi-probability for its alternatives such as:

$$P(D_i|Par(D_i)) = \frac{1}{|dom(D_i)|} \tag{1.8}$$

where $dom(D_i)$ describes the set of alternatives for each decision.

The transformation of value nodes consists on transforming them into a binary chance node (T/F) and then quantifying these nodes as follows, $\forall Par(V) \in Par(V)$:

$$P(v = T|Par(V)) = \frac{U(Par(V)) + K_2}{K_1} \tag{1.9}$$

where $K_1 = max(U(pa(V)))$ - $min(U(Par(V)))$
and $K_2 =$ - $min(U(pa(V)))$
Thus the probability function assigned to the value node where its value is false equals to:

$$P(v = F|Par(V)) = 1 - [\frac{U(Par(V)) + K_2}{K_1}]. \tag{1.10}$$

After the graphical transformation, the problem of evaluating influence diagrams is reduced to Bayesian inference. Cooper [4] describes this process for single decision problems (i.e. the initial influence diagram has just one decision node) and for multiply decision problems (i.e. the initial influence diagram has many decision nodes). While this method is applied for probabilistic influence diagrams, so the decision criterion will be MEU for both cases.

In the case of single decision problems with a decision node D_m and a set of evidence E, the optimal strategy can be computed by:

$$MEU(D_m, E) = \max_{D_m}[\sum_{Par'(V)} u(Par(V))P(Par'(V)|E, D_m))] \tag{1.11}$$

where Par'(V) is the set of chance nodes in the set (Par(V)) parents of the node V.

In the case of multiple decision problems with a set of decision nodes D = $\{D_1, .., D_m\}$ and a set of evidence E, the optimal strategy can be computed by:

$$MEU(D, E) = K_1 * \max_{d_{ij}}[P(v = T|d_{ij}, E)] - K_2 \tag{1.12}$$

Example 1.4. *Figure 1.4 presents the Bayesian network corresponding to the transformation of the influence diagram presented in figure 1.2. Tables 1.4 and 1.5 are obtained after transforming decision nodes D_1 and D_2 using equation 1.8. The table 1.6 presents the quantification of the value node performed using equation 1.9 after computing K_1 and K_2 such as:*
K_1 = U_{max} - U_{min} = 10 - 0 = 10
K_2 = - U_{min} = 0

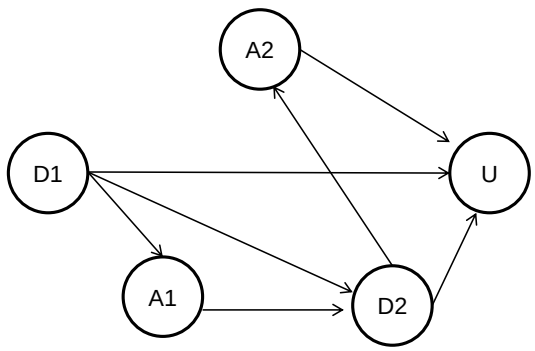

Figure 1.4: The Bayesian network corresponding to the influence diagram in figure 1.2

D_1	$\mathbf{P}(D_1)$
T	0.5
F	0.5

Table 1.4: Conditional probability of D_1

Example 1.5. *For computation of the optimal strategy relative to the influence diagram presented in figure 1.2, we perform inference in Bayesian network presented in figure 1.4 with use of equation 1.12. The obtained optimal strategy using MEU criterion is similar to the one obtained with decision trees which is: δ^* = {(D_2 = T), (D_1 = T)} = 5,5.*

1.3.3.3 Transformation of influence diagrams into junction trees

Since, the classical variable elimination and message passing schema are relevant only for inference in singly connected graphs; the junction tree algorithm was proposed [18, 19] to remedy the problem of inference in multiply connected graphs.

D_2	D_1	A_1	P(D_2|D_1,A_1)
T	T	T	0.5
T	T	F	0.5
T	F	T	0.5
T	F	F	0.5
F	T	T	0.5
F	T	F	0.5
F	F	T	0.5
F	F	F	0.5

Table 1.5: Conditional probability of D_2

D_1	D_2	A_2	P(V=T|D_1,D_2,A_2)	P(U=F|D_1,D_2,A_2)
T	T	T	1	0
T	T	F	0.1	0.9
T	F	T	0.5	0.5
T	F	F	0.5	0.5
F	T	T	0.2	0.8
F	T	F	0.7	0.3
F	F	T	0	1
F	F	F	0	1

Table 1.6: The conditional probability of value node V

This structure allows the representation of the problems by a set of interconnected cliques which include a certain number of variables.

Junction tree algorithm is an efficient inference algorithm which allows probability updating in multiply connected graphs with improvement of older methods of calculation where the joint probability table increases exponentially with the number of variables [2, 20]. Jensen et al. [13] take advantage of this method to evaluate influence diagrams.

In the following we will detail this process by carrying out the graphical transformation of an influence diagram into a junction tree and then we will apply the appropriate inference algorithm. This approach constitutes the basis of our proposal (Chapter 3).

The graphical transformation process can be performed by applying the algo-

rithm 1.2.

Algorithm 1.2: Construction of the strong junction tree

Data: In: An influence diagram
Result: A strong junction tree
begin
- Remove Informational arcs
- Moralization of the graph
- Remove utility nodes
- Triangulation if the moral graph
- Construction of the strong junction tree

end

The main operations mentioned in this algorithm can be defined such as:

1. ***Moralization of the graph***: Consists on marrying all parents sharing a common child. Namely, connect all nodes that are predecessors of the same node (even value node) with unoriented arcs. Then we remove the arrows direction of initial arcs.

2. ***Triangulation of the graph***: Involves eliminating variables from moral graph in a specific order which corresponds to reverse of the partial order of the influence diagram. While the order of elimination is imposed, this operation is called *strong triangulation*. The result of this process allows the obtention of clusters and separators that will be used to construct the strong junction tree.

3. ***Construction of the junction tree***: Performed by arranging the obtained clusters and separators as a connected tree where the strong root is the last cluster formed during the strong triangulation. The constructed tree, may be formed by one or several branches.

Example 1.6. *Figure 1.5 represents the result of the graphical manipulation of the initial diagram presented in figure 1.2. The strong junction tree presented in this figure is formed by two clusters:* $A_2D_1D_2$ *which is the root and the cluster* A_1D_1*. These clusters are separated with separator* D_1*.*

After the construction of the junction tree, we proceed to the propagation process in order to perform the computation task relative to evaluation of initial influence diagram. For this purpose, we have to deal with two type of potentials

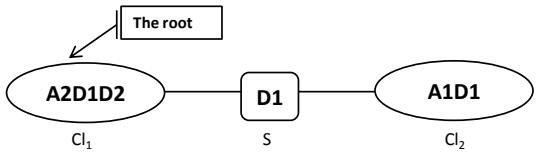

Figure 1.5: The strong junction tree corresponding to the initial influence diagram presented in figure 1.2

which are probability potential Φ_{Cl} and utility potential Ψ_{Cl} associated to each cluster. So, for the whole junction tree these potentials are defined as follows:

$$\Phi = \prod_{Cl \in Cl-set} \Phi_{Cl} \tag{1.13}$$

$$\Psi = \sum_{C \in Cl-set} \Psi_{Cl} \tag{1.14}$$

where Cl denotes a cluster and Cl-set denotes the set of all clusters.

Evaluation of influence diagrams amounts to the computation of the product of these two potentials (i.e. Φ and Ψ) relative to the obtained junction tree. The *absorption* in strong junction trees is quite similar to the principle of inference in classical junction tree algorithm [20], and it's based on variable elimination process. The principle of this method was introduced in [13] and it can be defined as follows:

First of all, we have to initialize the potentials of each cluster such as Φ_{Cl} equals to the product of conditional probability associated to each random variable included in this cluster, and Ψ_{Cl} corresponds to the sum of utilities associated to variables included in this cluster and were initially direct predecessors of value node.

For clusters without probability potential we assign the unit function and for those without utility potential we assign null function [13]. Also, these functions (i.e. unit function and null function) are associated respectively to initialize the separators potentials Φ_S and Ψ_S.

The potential of each variable should be associated to just one cluster. Therefore, if a variable occurs in several clusters its potential should be associated to the closest cluster to the leafs, containing this variable with its parents.

After performing the initialization, we proceed to the absorption that defines messages exchange between clusters. These messages are propagated from leafs to the strong root via separators and engender the potential updating for each cluster or separator that receives a message such as:

$$\Phi_{Cl_i}^{new} = \Phi_{Cl_i} * \Phi_S \tag{1.15}$$

$$\Psi_{Cl_i}^{new} = \Psi_{Cl_i} + \frac{\Psi_S}{\Phi_S} \tag{1.16}$$

where

$$\Phi_S = \sum_{Cl_j \setminus S}{}^{*} \Phi_{Cl_j} \tag{1.17}$$

$$\Psi_S = \sum_{Cl_j \setminus S}{}^{*} \Phi_{Cl_j} * \Psi_{Cl_j} \tag{1.18}$$

The notation $\sum_{Cl}^{*}$ is a generalized marginalization operator performed during the absorption [2]. Namely, when a cluster Cl_j sends a message to Cl_i through the separator we apply the marginalization for variables which are eliminated (i.e. variables contained in Cl_j and not in the separator). This marginalization depends on the variable's type. In the case of a chance variable C_i, the marginalization refers to the *sum* operator while for a decision node D_i it refers to the *max* operator.

The absorption is applied successively through the different branches of the tree. So, it ensures the updating of probability and utility potentials for all variables included in the junction tree clusters. By the end of this operation, we can determinate the optimal policy for every decision variable from the root or closest clusters containing this decision variable. Also, we can deduce the value of the best strategy after completing message passing by applying this equation:

$$\delta_i^* = \arg\max_{D_i} \sum_{dom(C_i) \setminus D_i} \Phi_{Cl_i} \Psi_{Cl_i}. \tag{1.19}$$

Example 1.7. *For the calculation of the optimal strategy using the junction tree presented in figure 1.5 and MEU as decision criterion, we have to compute the potentials updating of:* Φ_S, Ψ_S, *then* Φ_{Cl_1} *and* Ψ_{Cl_1} *using respectively equations: 1.17, 1.18, 1.15 and 1.16. So, we obtain the same result as previous methods:* δ^* *=* $\{(D_2 = T), (D_1 = T)\}$ *= 5,5.*

1.4 Conclusion

In this chapter, we have defined classical influence diagrams, which are probabilistic models having different evaluation methods. These methods can be classified into direct and indirect methods. Direct methods offer a direct simplification of the diagram structure to evaluate it. While, indirect methods ensure the evaluation by

transition into an intermediate structure which can be: a decision tree, Bayesian network, a junction tree or a decision circuit.

Chapter 2

Possibilistic influence diagrams

2.1 Introduction

As detailed in chapter 1, influence diagrams are powerful tools for representing and analyzing decision problems under uncertainty. These models were presented in a probabilistic context, where probability theory was the only framework to represent uncertainty and imprecision that characterize real word. However, it was proved that this theory is appropriate only when all numerical information are available. Therefore, probability theory presents some limits in regards to representation of total ignorance and qualitative uncertainty.

Possibility theory presents one of the non classical models that have been proposed to overcome the limits of probability theory. It offers a natural and simple model to handle qualitative and quantitative uncertain information. The development of this theory, has spawned the emergence of new decision models which are possibilistic influence diagrams.

These models, which benefit from the structural simplicity of classical influence diagrams and the efficiency of possibility theory to deal with uncertainty, are the subject of this chapter which is organized as follows: In section 2.2 we will present some basic elements of possibility theory and possibilistic decision criteria. In section 2.3 we will focus on the definition of possibilistic influence diagrams. Finally, section 2.4 is devoted to existing methods to evaluate possibilistic influence diagrams.

2.2 Basics on possibility theory

Possibility theory is a non-classical framework to handle uncertainty, introduced by Zadeh [33] and then developed by Dubois and Prade [9, 10].

2.2.1 Possibility distribution

The possibility distribution concept, is the core of possibility theory that encodes our knowledge on the real world. It's, denoted by π and expresses a possibility degree assigned to each element ω of the universe of discourse Ω. This possibility degree corresponds to a mapping from Ω to a bounded linearly ordered scale L, exemplified by $[0, 1]$.

The possibilistic scale L can be interpreted in two manners:
- In ***numerical*** or ***quantitative*** interpretation, the values of the possibility distribution make sense.
- In ***ordinal*** or ***qualitative*** interpretation, the possibility degree reflect only an ordering between the possible values.

Table 2.1: Possibility distribution π

$\pi(\omega) = 0$	ω is impossible
$\pi(\omega) = 1$	ω is totally possible
$\pi(\omega) > \pi(\omega')$	ω is preferred to ω' (or is more plausible)

In possibility theory, the extreme cases of knowledge are represented as follows:
- ***Complete knowledge*** : $\exists \omega_0,\ \pi(\omega_0) = 1$ *and* $\forall \omega \neq \omega_0,\ \pi(\omega) = 0$.
- ***Total ignorance*** : $\pi(\omega) = 1,\ \forall \omega \in \Omega$.

The main notions relative to possibility distribution are:

- ***Normalization***: A possibility distribution π is said to be *normalized* if there exists at least one element of Ω which is totally possible.

$$\exists \omega \in \Omega \ s.t \ \Pi(\omega) = 1 \tag{2.1}$$

- ***Marginalization***: Given a joint possibility distribution π on Ω. We can derive marginal distribution relative to subsets of variables using the *maximum*

operator.

$$\forall x \in Dom(X),\ \pi(x) = \max_{\omega \in \Omega}\{\pi(\omega) : \omega[X] = x\} \tag{2.2}$$

- ***Combination***: Given n possibility distributions $\{\pi_1, ..., \pi_n\}$ on $\{\Omega_1, ..., \Omega_n\}$ the combination of these distributions is performed depending on the interpretation of the possibilistic scale. We use *product* operator in the numerical case and the *minimum* operator in the ordinal case.

2.2.2 Possibility and necessity measures

In probability theory, uncertain knowledge about any state ψ is represented by a single probability measure P. This measure is dual i.e. we can deduce the probability degree assigned to ψ from its complement ($\neg\psi$): $P(X) = 1 - P(\neg\psi)$.

Contrary to probability theory, possibility theory is characterized by the use of two dual measures to describe uncertainty: the possibility measures $\Pi(\psi)$ and necessity measure $\mathrm{N}(\psi)$.
Possibility measure Π: Given a possibility distribution π, the possibility measure of any subset $\psi \subseteq \Omega$ is defined by:

$$\Pi(\psi) = \max_{\omega \in \psi} \pi(\omega). \tag{2.3}$$

$\Pi(\psi)$ denotes the possibility degree evaluating at which level ψ is **consistent** with the knowledge representation π.
Necessity measure N: Given a possibility distribution π, the necessity measure of any subset $\psi \subseteq \Omega$ is defined by:

$$N(\psi) = 1 - \Pi(\neg\psi) = \min_{\omega \in \psi} (1 - \pi(\omega)) \tag{2.4}$$

$\mathrm{N}(\psi)$ denotes the necessity degree evaluating at which level ψ is **certainly** implied by the knowledge representation π.

2.2.3 Possibilistic conditioning

The conditioning is performed in the case of independence relationship. This notion consists on updating an initial possibility distribution π by π' after observation of new certain evidence e. So, π is replaced by $\pi' = (.|\psi)$ with $\psi \neq 0$ and $\Pi(\psi) > 0$.

According to the possibilistic scale interpretation, the definition of conditioning respectively for both numerical and ordinal cases is given by:

- *Product* based conditioning:

$$\pi(\omega \mid_p \psi) = \begin{cases} \frac{\pi(\omega)}{\Pi(\psi)} & \text{if } \omega \in \psi \\ 0 & otherwise. \end{cases} \tag{2.5}$$

- *Min* based conditioning:

$$\pi(\omega \mid_m \psi) = \begin{cases} 1 & if\ \pi(\omega) = \Pi(\psi) \text{ and } \omega \in \psi \\ \pi(\omega) & if\ \pi(\omega) < \Pi(\psi) \text{ and } \omega \in \psi \\ 0 & otherwise. \end{cases} \tag{2.6}$$

These definitions for both interpretations should satisfy:

$$\forall \omega,\ \pi(\omega) = \pi(\omega|\psi) \otimes \Pi(\psi). \tag{2.7}$$

where $\otimes$ is the product operator in the numerical case and the min operator in the ordinal case.

2.2.4 possibilistic decision criteria

In probabilistic context, solving a decision problem is generally done on the basis of maximal expected utility (MEU). So, this decision criterion was appropriate to evaluate probabilistic influence diagrams seen in chapter 1. With the development of possibility theory, there was a need to find the possibilistic counterpart of this criterion that would be used to evaluate possibilistic influence diagrams (section 2.4).

In this regard, several researches were devoted to study this problem, and they have given rise to panoply of decision criteria, which are defined with consideration of the used possibilistic scale such as:

- For *qualitative* possibilistic scale we can define: pessimistic (U_{pes}) and optimistic (U_{opt}) utilities [12] and binary utilities (PU) [16].
- For both *numerical or qualitative* possibilistic scale we can define possibilistic likely dominance (LN, LΠ) [7], order of magnitude expected utility (OMEU) [17] and possibilistic choquet integrals (Ch_π,Ch_N).

We note that possibilistic choquet integrals (Ch_π,Ch_N) is the only decision criterion adequate to evaluate heterogeneous influence diagrams (section 2.3).

In the following, we will give more details about pessimistic utility, optimistic utility and order of magnitude expected utility which will be used later in our

approach. For qualitative influence diagrams we will use pessimistic and optimistic utilities and for quantitative influence diagrams we will use the order of magnitude expected utility.

2.2.4.1 Optimistic and pessimistic utilities

Pessimistic and optimistic utilities [11, 12] present the counterpart of the expected utility in the possibilistic framework. They are based on the behavior of decision maker (an optimistic or pessimistic person).

These utilities are used to evaluate different decision policies in a purely qualitative context (i.e. utilities and possibilities are expressed in an ordinal scale). ***Pessimistic utility*** : Introduced by Whalen [8] and it defines the worst decision policies relatively plausible. The pessimistic utility denoted by (U_{Pes}), is computed as follows:

$$U_{pes}(d) = \min_{x \in X} \max\ (1 - \pi(x), u(d(x))) \tag{2.8}$$

Optimistic utility : Introduced by Yager [8] and it defines the best decision policies the most plausible. The optimistic utility denoted by (U_{opt}), is computed as follows:

$$U_{opt}(d) = \max_{x \in X} \min\ (\pi(x), u(d(x))) \tag{2.9}$$

2.2.4.2 Order of magnitude expected utility

Order of magnitude expected utility was initially proposed by [32]. It presents a quantitative or qualitative representation of possibility degrees. It expresses a degree of "dissatisfaction" for a given event. So, u(x) $<$ u(x') means that x is more desirable than x'. The order of magnitude expected utility can be computed by:

$$OMEU(d) = \min_{x \in X} \{\pi(x) + u(x)\} \tag{2.10}$$

2.3 Possibilistic influence diagrams

Further to the evolution of possibility theory, there was emergence of new variants of graphical models, such as possibilistic influence diagrams. They, are denoted by $\Pi\mathcal{ID}^{u}_{\otimes}$ and present an alliance between graph theory and possibility theory. These models exhibit the same graphical component as classical (probabilistic) influence diagrams presented in section 1.2, but the quantification is different. Indeed, the

structure of possibilistic influence diagrams is constituted by a set of nodes N, which is divided into three subsets : Chance nodes C, decision nodes D and value nodes V, and a set of arcs A, that may be informational or conditional arcs.

Example 2.1. *Let us consider the figure 2.1 to represent the graphical component of a possibilistic influence diagram. The outlined structure is defined by two chance nodes* (A_1, A_2)*, two decision nodes* (D_1, D_2) *and a value node V.*

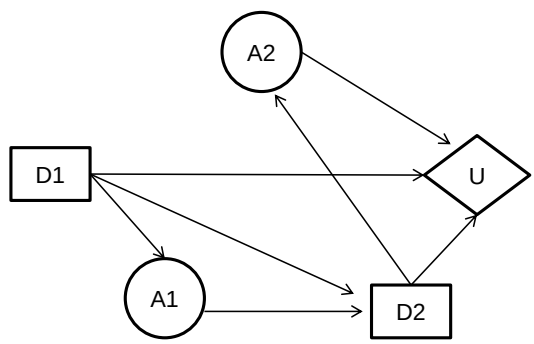

Figure 2.1: Example of graphical component of a possibilistic influence diagram

The quantification of $\Pi\mathcal{ID}^u_{\otimes}$ is ensured by assigning possibility degrees $\Pi(C_i)$ to chance nodes and utilities $\mathrm{U}(V_i)$ to value nodes. These values can be assigned independently on a numerical or ordinal possibilistic scale. Namely, the set of chance nodes C, can be defined in numerical or ordinal setting, as well as the set of value node which can be in the same setting of chance nodes or not.

These different combinations of quantification allow the definition of several types of diagrams that can be classified into [15] :

1. ***Homogeneous possibilistic influence diagrams*** where chance nodes and value nodes are quantified in the same setting such as:

   - *Product-based* possibilistic influence diagrams, denoted by $\Pi\mathcal{ID}^*_*$ where both chance nodes and value nodes are quantified according to a numerical possibilistic scale.
   - *min-based* possibilistic influence diagrams, denoted by $\Pi\mathcal{ID}^{min}_{min}$ where both chance nodes and value nodes are quantified according to an ordinal possibilistic scale.

We will focus on such diagrams (i.e. $\Pi\mathcal{ID}^{min}_{min}$ and $\Pi\mathcal{ID}^{*}_{*}$) for our proposal.

2. ***Heterogeneous possibilistic influence diagrams*** where chance nodes and value nodes are not quantified in the same setting. If chance nodes are quantified in numerical setting, the value nodes are quantified in ordinal setting and vice versa. These variants are denoted respectively by $\Pi\mathcal{ID}^{min}_{*}$ and $\Pi\mathcal{ID}^{*}_{min}$.

In the following, table 2.2 summarize the different variants of possibilistic influence diagrams.

U/Π	**Ordinal**	**Numerical**
Ordinal	$\Pi\mathcal{ID}^{min}_{min}$	$\Pi\mathcal{ID}^{min}_{*}$
Numerical	$\Pi\mathcal{ID}^{*}_{min}$	$\Pi\mathcal{ID}^{*}_{*}$

Table 2.2: Variants of possibilistic influence diagrams

Since possibilistic influence diagrams possess the same graphical component than classical ones, so they maintain the same properties of the $\mathcal{DAG}$ that encodes the dependency relationships between variables. This allows us to conclude that for each instance c_{ij} of a chance node C_i we assign a conditional possibility degree $\pi(c_{ij}|Par(C_i))$ in the context of its parents.

As mentioned in section 2.2, conditional possibility distributions of possibilistic influence diagrams may be numerical, so we obtain $\Pi\mathcal{ID}^{u}_{*}$ or they may be ordinal so we talk about $\Pi\mathcal{ID}^{u}_{min}$.
For both cases, the conditional possibility distribution should be normalized such as:

$$\max_{c_{ij}\in Dom(C_i)} \Pi(c_{ij}|Par(C_i)) = 1 \tag{2.11}$$

Same as chance nodes, utility nodes are defined in the context of their parents. Their values can be numerical, so we have $\Pi\mathcal{ID}^{*}_{\otimes}$; or ordinal so we deal with $\Pi\mathcal{ID}^{min}_{\otimes}$. For decision nodes, there are no quantification assigned to them.

At this stage, based on the quantification presented above and the basics of classical influence diagrams seen in chapter 1, the following chain rule can be defined for possibilistic influence diagrams [14] :

$$\Pi(C|D) = \bigotimes_{C_i\in C} \Pi(C_i|Par(C_i)) \tag{2.12}$$

where $\otimes$ is the min operator in the case of $\Pi\mathcal{ID}^{u}_{min}$ and the product operator in the case of $\Pi\mathcal{ID}^{u}_{*}$.

Example 2.2. *Let us consider tables: 2.3, 2.4 and 2.5 to quantify possibilistic influence diagram presented in figure 2.1.*

A_1	D_1	$\Pi(A_1\|D_1)$
T	T	1
T	F	0.4
F	T	0.2
F	F	1

Table 2.3: Conditional possibility of A_1

A_2	D_2	$\Pi(A_2\|D_2)$
T	T	0.3
T	F	1
F	T	1
F	F	0.4

Table 2.4: Conditional possibility of A_2

D_1	D_2	A_2	$u(D_1, D_2, A_2)$
T	T	T	0.2
T	T	F	0.3
T	F	T	0.4
T	F	F	0.6
F	T	T	1
F	T	F	0
F	F	T	0.1
F	F	F	0.7

Table 2.5: Utility function of (D_1, D_2, A_2)

2.4 Existing methods for evaluating possibilistic influence diagrams

After defining possibilistic influence diagrams, we proceed to the most important task for our research which is the evaluation of these structures. This amounts to

determine the optimal strategy δ^* of the outlined problem described with consideration to a possibilistic decision criterion (section2.2.4) adequate to the corresponding $\Pi\mathcal{ID}^u_{\otimes}$.

In [15], authors present a clarification of the decision criteria used for each type of possibilistic influence diagrams:

$\Pi\mathcal{ID}/U$	U_{pes}	U_{opt}	PU	LN	$L\Pi$	OMEU	Ch_N	Ch_Π
$\Pi\mathcal{ID}^{min}_{min}$	*	*	*	*	*	*	*	*
$\Pi\mathcal{ID}^{*}_{*}$				*	*	*	*	*
$\Pi\mathcal{ID}^{min}_{*}$							*	*
$\Pi\mathcal{ID}^{*}_{min}$							*	*

Table 2.6: Correspondence between possibilistic decision criteria and different types of possibilistic influence diagrams

Although the literature is rich of research works that treat the problem of evaluating influence diagrams, there are just few of them which are dedicated to evaluate possibilistic influence diagrams. We can mention, in particular, Garcia et al. [23] that propose transformation of initial influence diagrams into possibilistic decision trees. This method provides only evaluation of qualitative influence diagrams (ΠID^{min}_{min}) using optimistic (U_{opt}) and pessimistic (U_{pes}) utilities. We can also cite [25] where authors provide a direct evaluation using variable elimination adequate to ΠID^{min}_{min} using order of magnitude expected utility (OMEU). Finally, we cite [14] which is a recent research that offers the possibility to evaluate possibilistic influence diagrams with all possible decision criteria through two methods based on transformation into possibilistic decision trees and possibilistic networks. These methods will be detailed respectively in what follows.

2.4.1 Transformation of possibilistic influence diagrams into possibilistic decision trees

The evaluation method presented below is a possibilistic adaptation [14] of the method of Howard and Matheson [1]. Since the quantification of the graph (possibilistic or probabilistic) did not affect its structure, then this adaptation resumes the same procedures for graphical transformation seen in section 1.3. These procedures allow the transformation of influence diagrams into decision networks and then into decision tree networks and finally into decision trees.

After construction of the decision tree, its quantification will be done according to the quantification of the initial diagram. So, the arcs incident from chance nodes will be labeled with possibility degrees, the arcs incident from decision nodes will be labeled with different alternatives, and the leafs will be quantified by utility functions.

To evaluate the obtained possibilistic decision trees, this method offers an adaptation of the classical algorithm for possibilistic framework. So, instead of using the dynamic programming with the MEU as decision criterion, this proposal brings changes to this algorithm (i.e. dynamic programming) to be adequate for possibilistic decision criteria seen in (section 2.2) except of Choquet integrals, where a branch and bound algorithm should be applied [14].

Example 2.3. *Figure 2.2 represents the decision tree corresponding to the transformation of influence diagram presented in figure 2.1. The quantification of this possibilistic decision tree resume the quantification of the initial diagram presented in tables: 2.3, 2.4 and 2.5. For this example we will suppose that the chosen decision criterion is the optimistic utility, and so we obtain the optimal strategy such as* $\delta^* = \{(D_1 = T/F), (D_2 = F)\} = 0{,}4$.

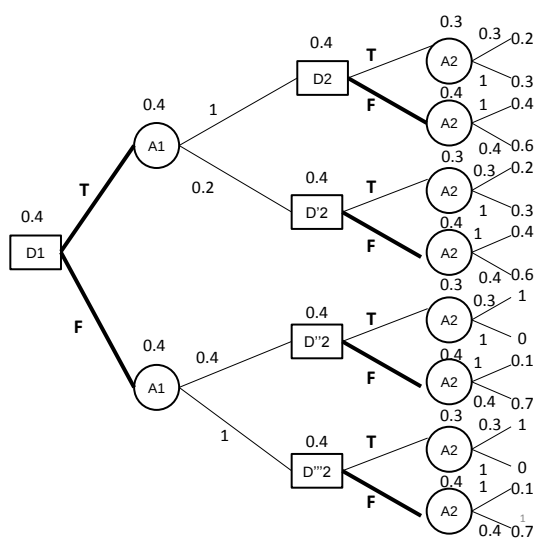

Figure 2.2: The possibilistic decision tree corresponding to the possibilistic influence diagram in figure 2.1 and its optimal strategy using U_{opt} criterion.

2.4.2 Transformation of possibilistic influence diagrams into possibilistic networks

The method presented in [14] was inspired from the Cooper's method [4] presented in section 1.3. So, it evaluates an influence diagram through possibilistic networks which present the possibilistic counterpart of Bayesian networks. Therefore, the author presents a pretreatment step that enriches the classical approach by the possibility of solving influence diagrams with several value nodes.

The pretreatment phase consists on reducing the number of value nodes to just one value node denoted by V_r, that inherits the parents of all value nodes. The value of V_r is obtained by:

$$u(V_r|Par(V_r)) = \min_{i=1...k} u(V_i|Par(V_r)) \tag{2.13}$$

For the transformation phase, decision nodes from initial diagram can be transformed into chance nodes with total ignorance such as:

$$\Pi(d_{ij}|Par(D_i)) = 1, \forall D_i \in D \tag{2.14}$$

Besides, value node V_r is transformed into binary chance node and its quantification is assigned according to the nature of initial utilities such as:

- If utilities and possibilities are commensurable and the uncertainty scale is [0,1], the quantification of V_r is defined by:

  $$\Pi(V_r = T|Par(V_r) = u(Par(V_r))) \tag{2.15}$$

  and

  $$\Pi(V_r = F|Par(V_r)) = 1 \tag{2.16}$$

- If utilities and possibilities are not commensurable, then utilities should be transformed into the scale [0,1] as follows:

  $$\Pi(V_r = T|Par(V_r) = \frac{u(Par(V_r)) - U_{min}}{U_{max} - U_{min}} \tag{2.17}$$

  and

  $$\Pi(V_r = F|Par(V_r)) = 1 \tag{2.18}$$

  where U_{max} and U_{min} are respectively the maximal and the minimal utilities in U(Par(V_r))

After these transformations and according to the quantification setting of the initial diagram $\Pi\mathcal{ID}^u_{\otimes}$, the adequate decision criterion should be chosen (section 2.2) and then the optimal strategy can be computed through inference in the obtained possibilistic network.

Example, for the case of the optimistic and pessimistic utilities, the optimal strategies can be computed respectively by:

$$U^*_{opt}(D_i, E) = \max_{D_i} \Pi(V_r = T|D_i, E) \tag{2.19}$$

$$U^*_{pes}(D_i, E) = \max_{D_i} \min_{Par'(V_r)} \Pi(V_r = T|Par'(V_r), D_i, E) \tag{2.20}$$

where Par'(V_r) is the set of chance nodes in the parents of value node V_r.

Example 2.4. *Figure 2.3 represents the possibilistic network corresponding to the transformation of influence diagram presented in figure 2.1. The quantification of this possibilistic network after transformation of the diagram is presented by tables: table 2.7, table 2.8 and 2.9.*

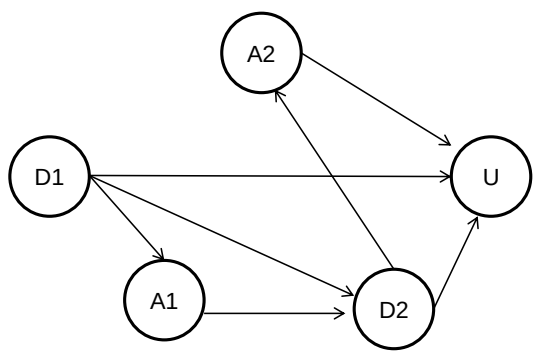

Figure 2.3: The possibilistic network corresponding to the possibilistic influence diagram in figure 2.1

D_1	$\Pi(D_1)$
T	1
F	1

Table 2.7: Conditional possibility of D_1

Example 2.5. *For the calculation of the optimal strategy corresponding to the possibilistic network in figure 2.3 using U_{opt} as decision criterion. We will first*

D_2	D_1	A_1	$\Pi(D_2 \mid D_1, A_1)$
T	T	T	1
T	T	F	1
T	F	T	1
T	F	F	1
F	T	T	1
F	T	F	1
F	F	T	1
F	F	F	1

Table 2.8: Conditional possibility of D_2

D_1	D_2	A_2	$\Pi(U = T \mid D_1, D_2, A_2)$	$\Pi(U = F \mid D_1, D_2, A_2)$
T	T	T	0.2	1
T	T	F	0.3	1
T	F	T	0.4	1
T	F	F	0.6	1
F	T	T	1	1
F	T	F	0	1
F	F	T	0.1	1
F	F	F	0.7	1

Table 2.9: The conditional possibility of V_r

determine the best policy for D_2 such as:
$\Pi(V = T/D_2 = T) = 0.3$ *and* $\Pi(V = T/D_2 = F) = 0.4$ *, so the best decision for* $D_2 = F$.
Then, we will determine the best policy for D_1 such as:
$\Pi(V = T/D_1 = T, D_2 = F) = 0.4$ *and* $\Pi(V = T/D_1 = F, D_2 = F) = 0.4$ *, so the best decision for* $D_1 = T$ *or* F.
Then, the optimal strategy is: $\delta^* = (D_1 = T/F, D_2 = F) = 0{,}4$.

2.5 Conclusion

In this chapter we have presented basic elements of possibility theory which has been proposed to fill the limits of probability theory. Besides, we have presented possibilistic decision criteria with focus to the ones that will be used in our proposal

in chapter 3. The rest of the chapter was dedicated to define possibilistic influence diagrams and to present existing methods to evaluate them. Among these methods we have detailed two indirect methods that allow the evaluation of the initial diagrams after transforming them into possibilistic decision trees or possibilistic networks.

Chapter 3

New approach to evaluate possibilistic influence diagrams

3.1 Introduction

As detailed in chapter 1, evaluation methods of classical influence diagrams can be classified into two main categories which are direct evaluation and indirect evaluation methods. While the direct methods require a lot of probabilistic calculations there have been development of indirect methods. These indirect methods consist on evaluating probabilistic influence diagrams after transformation into a secondary structure which can be: decision trees [1], Bayesian networks [4], junction trees [13] or decision circuits [3].

In chapter 2, we have mentioned that these classical influence diagrams, based on probability theory, present some limits to represent uncertainty. So, there have been emergence of possibilistic influence diagrams. In the same chapter we have presented an adaptation of two indirect methods to evaluate possibilistic influence diagrams.

This chapter, will be dedicated to detail a possibilistic adaptation of Jensen's method, that evaluates influence diagrams using strong junction trees [13]. This new approach offers the possibility to evaluate possibilistic influence diagrams quantified with consideration to both: ordinal or numerical possibilistic scale. The main sections of this chapter are: Section 3.2 is devoted to the graphical transformation of initial possibilistic influence diagrams into possibilistic strong junction trees.

Section 3.3 introduces the evaluation process of respectively min-based possibilistic influence diagrams (subsection 3.3.1) and product-based possibilistic influence diagrams (subsection 3.3.2). Section 3.4 presents a comparison of our method with existing methods to evaluate possibilistic influence diagrams. Finally, section 3.5 presents the global architecture of our implementation.

3.2 Graphical transformation

As indicated previously, the evaluation of an influence diagram amounts to performing a sequence of decisions that maximize utility. These decisions should be taken with consideration to a set of observed variables in the moment of each decision making. This definition evokes the notion of decision windows (section 1.3.1) and allows the description of the evaluation problem for $\Pi\mathcal{ID}$s by:

$$\delta_n^* = \max_{D_n} \sum_{Sc_n} \Pi(SC_n | SC_0, ..., Sc_{n-1}, D_1, ..., D_n) \tag{3.1}$$

Our idea here is to evaluate possibilistic influence diagrams by transforming them into possibilistic strong junction trees which is an adaptation of the probabilistic approach seen in chapter 1.

The principle of our proposal is similar to the probabilistic case especially for the graphical manipulation, since the construction of the strong junction tree is totally independent of the numerical values. For this process, we will detail the same approach presented in section 1.3.3.3, and it will be applied for product-based possibilistic influence diagrams and also min-based possibilistic influence diagrams.

According to a well known result in graph theory, triangulation offers the possibility to eliminate variables from the $\mathcal{DAG}$ without adding edges. This property allows to not add new constraints to diagrams. But in practice, to be triangulated, the graph should be moral. The moralization of a given graph consists on marrying parents of a common node (i.e. adding new arcs), and then removing the direction of existing arcs.

Formally, for a given $\Pi\mathcal{ID}$, the corresponding moral graph $\mathcal{MG}$ can be obtained through performing the algorithm 3.1.

Algorithm 3.1: Constructing of moral graph

Data: Initial $\Pi\mathcal{ID}$
Result: Moral graph $\mathcal{MG}$
begin
- Remove informational arcs
- Connect the parents set of each node in the $\Pi\mathcal{ID}$ by adding edges
- Remove arrows direction
- Remove value nodes(s)

end

Example 3.1. *Let us continue with the same possibilistic influence diagram represented in figure 2.1. The corresponding moral graph $\mathcal{MG}$ is presented in figure 3.1. The moralization consists on removing informational arcs (i.e. arcs between D_1 and D_2, and between A_1 and D_2), adding edges for parents of common child (i.e. edge between D_1 and A_2, and edge between D_1 and D_2, which are parents of V) and then removing arrows direction and the value node V.*

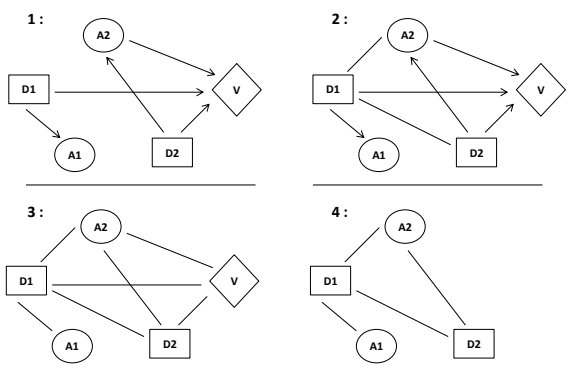

Figure 3.1: Moral graph relative to the possibilistic influence diagram presented in figure 2.1

The triangulation process is performed to create clusters of junction trees and to avoid cycles, if they exist, in the moral graph. The task of finding an optimal triangulation for any given graph is stated as NP-Hard problem but there are several heuristics to solve it. For $\mathcal{ID}$s, this task is driven according

to reverse of the temporal order, hence the name of *strong triangulation*. This elimination order allows the reduction of the complexity.
The algorithm 3.2 describes the strong triangulation process for $\Pi\mathcal{ID}$s:

Algorithm 3.2: Triangulation of the moral graph

Data: Moral graph $\mathcal{MG}$
Result: Triangulated moral graph $\mathcal{MG}'$, Cluster_set, Separator_set
begin
- Let ***elimination order*** be a vector containing nodes in reverse of the temporal order.
- Let ***Index_*** Cl_j an index attributed to each formed cluster.
- Let ***Index_*** S_j an index attributed to each formed separator.

$Cluster_set \leftarrow \emptyset$;
$Separator_set \leftarrow \emptyset$;
$\mathcal{MG}' \leftarrow \mathcal{MG}$;
while *there exists nodes in* $\mathcal{MG}'$ **do**
  $A \leftarrow$ elimination order [i];
  if *the adjacents of A in* $\mathcal{MG}'$ *are not connected* **then**
    - Add edges to connect all of its adjacent nodes;
  - Form a cluster Cl_j containing A and its adjacent nodes;
  - Delete A from $\mathcal{MG}'$;
  if *there exists a node B in the adjacents of A that has only adjacents in* Cl_j **then**
    - Connect the adjacents in $\mathcal{MG}'$ of each node in $Cl_j \backslash B$;
    if *B still has adjacents only in* Cl_j **then**
      - Delete B from $\mathcal{MG}'$;
  - Associate to Cl_j the separator S_j with the remaining nodes in Cl_j;
  - $Index_Cl_j \leftarrow (Index_{Cl_j} + number\ of\ removed\ nodes)$;
  - $Index_S_j \leftarrow Index_{Cl_j}$;
  - $Cluster_set \leftarrow Cluster_set \cup \{Cl_j\}$;
  - $Separator_set \leftarrow Separator_set \cup \{S_j\}$;
end

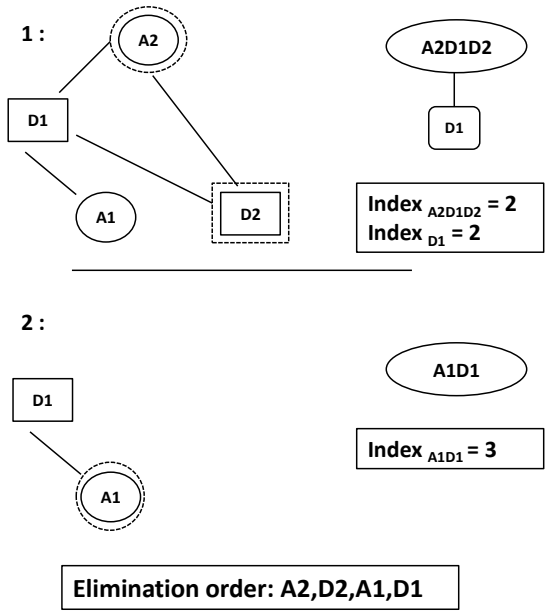

Figure 3.2: Strong triangulation of the moral graph presented in figure 3.1

Example 3.2. *Figure 3.2 presents the triangulated graph obtained after performing the strong triangulation process on the moral graph of figure 3.1. This figure shows the obtained clusters (i.e. cluster $A_2D_1D_2$ and cluster A_1D_1) as well as separator D_1 associated to the first cluster.*

Using $Cluster_set$, $Separator_set$ and their indexes (i.e. $Index_Cl_j$, $Index_S_j$) we can construct the so-called ***Possibilistic strong junction tree*** ($\Pi\mathcal{SJT}$) as follows:
Every separator in $Separator_set$ which is already connected to a cluster, will be connected to another cluster from $Candidate_clusters$.
$Candidate_clusters$ is a subset of $Cluster_set$ containing clusters with indexes superior to the separator index. Besides, the chosen cluster should contain all variables of the separator that will be connected to it. This procedure is given by algorithm 3.3:

Algorithm 3.3: Building an optimal possibilistic junction tree

Data: Cluster_set, Separator_set, Index_Cl_i, Index_S_i
Result: Possibilistic Strong Junction tree
begin
  while *there exists a cluster* $Cl_i \in Cluster_set$ **do**
    - Select a separator S_i from Separator_set;
    - $Candidate_Clusters \leftarrow$ Clusters where $Index_Cl_j > Index_S_i$;
    - Choose a cluster from Candidate_Clusters where $S_i \subset Cl_j$;
    - Connect S_i to Cl_j;
    - $\Pi\mathcal{SJT} \leftarrow \Pi\mathcal{SJT} \cup \{Cl_i, S_i, Cl_j\}$;
    - Remove Cl_i and Cl_j from Cluster_set;
end

This procedure provides the possibilistic strong junction tree corresponding to the initial diagram. This structure, presents a tree with one or many branches, where the clusters constitute the tree nodes. Generally, the last cluster formed during the strong triangulation represents the root.

Example 3.3. *Figure 3.3 represents the result of the graphical transformation performed on the initial diagram presented in figure 2.1. The possibilistic strong junction tree presented in this figure is formed by two clusters:* $A_2D_1D_2$ *which is the root and* A_1D_1*. These clusters are separated with the separator* D_1*.*

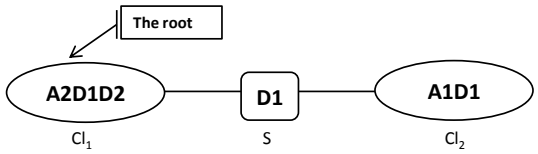

Figure 3.3: The possibilistic strong junction tree corresponding to the initial influence diagram presented in figure 2.1

3.3 Possibilistic absorption

Once the construction of the possibilistic strong junction tree is performed, we carry on to the *absorption* process which consists on a particular propagation, that allows the inference among clusters with marginalization operations. Thus, the evaluation problem is reduced to inference in possibilistic strong junction trees.

In order to do this, we assign for each cluster Cl_i (resp. separator S_i) of the $\Pi\mathcal{SJT}$ two local potentials which are *possibility potential* and *utility potential*. The first one defines the possibility joint distribution associated to each cluster and separator denoted by Φ_{Cl_i}(resp. Φ_{S_i}) and the other one refers to the utility degree relative to each cluster and separator denoted by Ψ_{Cl_i} (resp. Ψ_{S_i}).

We recall that the possibility theory offers two possibilistic scales which are numerical scale and ordinal scale (section 2.2.1). Therefore, the quantification of possibilistic influence diagrams gives rise to different variants of diagrams (section 2.3). These variants depend on the nature of quantification associated to chance nodes and value nodes. In our case, we will focus on homogeneous influence diagrams where both chance nodes and value nodes are either numerical or qualitative.

3.3.1 Absorption in min-based possibilistic strong junction trees

The *min-based possibilistic strong junction tree* ($\Pi\mathcal{SJT}^{min}_{min}$) is the result of graphical transformation performed on a min-based possibilistic influence diagram (i.e. chance and value nodes are quantified qualitatively). The optimal strategy relative to the initial influence diagram can be obtained by performing absorption on the $\Pi\mathcal{SJT}^{min}_{min}$. This absorption is based on message passing mechanism which allows the updating of clusters and separators potentials and it's carried out in two phases: ***Initialization*** and ***Global absorption***.

3.3.1.1 Initialization in min-based possibilistic strong junction trees

The initialization of any $\Pi\mathcal{SJT}_{min}^{min}$ consists on quantifying this structure with initial possibility distributions and utility functions, and it is performed as follows:
We assign a possibility potential for each cluster that corresponds to its local joint distribution, which is computed from the initial diagram.

Proposition 3.1. *For each cluster Cl_i we assign a possibility potential satisfying:*

$$\Phi_{Cl_i} = \Pi_m(Cl_i) \tag{3.2}$$

where $\Pi_m = \min_{j:1..m} \Pi(C_{ij}|Par(C_{ij}))$, where C_{ij} is a chance node C_j in the cluster Cl_i.

If chance node C_j exists in more than cluster, its possibility degree $\Pi(C_j|Par(C_j))$ should be assigned to one and only one clique Cl_i, which is the closest to the leaf, where $dom(C_j) \subseteq dom(Cl_i)$.

Also, we assign an utility potential to each cluster which corresponds to an ordinal satisfaction towards the different strategies. This potential is computed from the utility function of the initial diagram.

Proposition 3.2. *For each cluster Cl_i we assign an utility potential satisfying:*

$$\Psi_{Cl_i} = \min_{i:1..n} U_i(Par'(V)) \tag{3.3}$$

where Par'(V) = Par (V) in Cl_i
and $U_i(Par'(V))$ is the utility relative to each chance or decision node in Cl_i which was a direct parent of the value node V_i.

For clusters (resp. separators) without possibility potential we assign the unity potential, and for clusters(resp. separators) without utility potential we assign the null utility potential.

The algorithm 3.4 defines the initialization process corresponding to $\Pi\mathcal{SJT}_{min}^{min}$.

Algorithm 3.4: Min-based Initialization

Data: Initial distributions, Initial utility functions, $\Pi\mathcal{SJT}^{min}_{min}$

Result: Initialized $\Pi\mathcal{SJT}^{min}_{min}$

begin

  foreach *cluster* Cl_i **do**

    $\phi^I_{Cl_i} \leftarrow 1$;

    $\psi^I_{Cl_i} \leftarrow 0$;

  foreach *separator* S_i **do**

    $\phi^I_{S_i} \leftarrow 1$;

    $\psi^I_{S_i} \leftarrow 0$;

  foreach *variable* $A \in C$ **do**

    - Choose a cluster Cl_i *(a leaf or a closer cluster) containing* $\{A\} \cup Par(A)$*:*

    $\phi^I_{Cl_i} \leftarrow \min(\phi^I_{Cl_i}, \Pi(A|Par(A)))$;

    foreach variable A $\in Par(V_i)$ **do**

      - Choose a cluster Cl_i *(a leaf or a closer cluster) containing* $\{A\} \cup Par(A)$*:*

      $\psi^I_{Cl_i} \leftarrow \max(\psi^I_{C_i}, U(A|Par(A)))$;

end

Example 3.4. *For this example we will consider that tables 2.3, 2.4 and 2.5 present a qualitative quantification of the influence diagram presented in figure 2.1. Figure 3.4 represents the initialization of the min-based strong junction tree presented in figure 3.3, where possibilities and utilities associated to clusters and separators are provided from the tables cited above.*

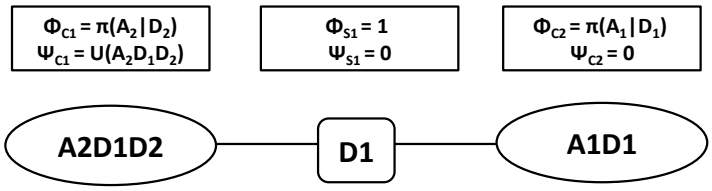

Figure 3.4: Initialization of the min-based strong junction tree

3.3.1.2 Global absorption in min-based possibilistic strong junction trees

The absorption or propagation in $\Pi\mathcal{SJT}_{min}^{min}$ is similar to the *collect process* in classical junction trees propagation algorithms. The absorption is performed from leafs to the root, and it's based on the marginalization operation. The latter presents the counterpart of variable elimination in the graphical manipulation.

Proposition 3.3. *For every removed node N_i (i.e. chance or decision node) in Cl_i, after marginalizing on N_i, the possibility distribution associated to this cluster will be updated as follows:*

$$\Phi_{Cl_i} = \max_{Cl_i \setminus N_i} \Phi_{Cl_i} \tag{3.4}$$

The global absorption is performed via message passing mechanism, where exchanged messages allow updating of clusters and separators potentials. This procedure should be performed after the choice of the decision criterion that would be used to evaluate the initial diagram. Among decision criteria presented in section 2.2, we are in particular interested to the most qualitative decision criteria which are optimistic (U_{opt}) and pessimistic (U_{pes}) utilities. The algorithm 3.5 allows the global absorption with consideration to these criteria.

The computation of equations mentioned in algorithm 3.5 amounts to updating of clusters and separators potentials. These potentials should be updated as follows:

- Conditional potential of the separator:

$$\Phi_{S_i} = \sum_{Cl_i \setminus S_i}{}^{*} \Phi_{Cl_i} \tag{3.5}$$

  where $\sum^*$ refers to the marginalization operator.

- Utility potential of the separator:
  It depends on the chosen decision criterion such as:

  - In the case of *optimistic decision criterion*:

$$\Psi_{S_i} = \max_{Cl_i \setminus S_i} \min\left(\Psi_{Cl_i},\ \Phi_{Cl_i}\right) \tag{3.6}$$

Algorithm 3.5: Min-based Global propagation

Data: Initialized $\Pi\mathcal{SJT}^{min}_{min}$
Result: Updated $\Pi\mathcal{SJT}^{min}_{min}$
begin
- Let ***Branch*** be an ordered vector containing clusters and their separators from leaf to the root.
- Let ***Decision-criterion*** be the chosen criterion to evaluate the initial diagram which can be U_{opt} or U_{pes}.

foreach *Branch in* ΠSJT^{min}_{min} **do**
  for i ← 1 to length(Branch)-1 **do**
    $Cl_i \leftarrow$ *Branch [i];*
    $S_k \leftarrow$ *Branch [i+1];*
    $Cl_j \leftarrow$ *Branch [i+2];*
    if Decision-criterion = U_{opt} **then**
      - Post a message from Cl_i to S_k using equations: 3.5 and 3.6;
    else
      - Post a message from Cl_i to S_k using equations: 3.5 and 3.7;
    - Post a message from S_k to Cl_j using equations: 3.8 and 3.9;

end

– In the case of *pessimistic decision criterion*:

$$\Psi_{S_i} = \min_{Cl_i \setminus S_i} \max\ (\Psi_{Cl_i},\ N(Cl_i)) \tag{3.7}$$

where N(Cl_i) equals to (1 - Φ_{Cl_i}).

- Conditional potential of the cluster:

$$\Phi^{new}_{Cl_j} = \min(\Phi_{Cl_j},\ \Phi_{S_i}) \tag{3.8}$$

- Utility potential of the cluster:

$$\Psi^{new}_{Cl_j} = \max\ (\Psi_{Cl_j},\ \frac{\Psi_{S_i}}{\Phi_{S_i}}) \tag{3.9}$$

For both optimistic and pessimistic cases, the optimal strategy δ^* is composed of a set of policies that maximize each decision. The optimal policy δ^*_i

for each decision node D_i can be inferred from the root or the closest cluster containing this decision node, such as:

$$\delta_i^* = \arg\max_{D_i} \Psi_{Cl_j} \tag{3.10}$$

Thus, the value of the optimal strategy for $\Pi\mathcal{SJT}_{min}^{min}$ is determined from the root after message passing and its value is the value that maximizes the utility potential of this cluster.

Example 3.5. *For the calculation of the optimal strategy of the min-based possibilistic strong junction tree presented in figure 3.3 using U_{opt} as decision criterion, we have to compute the potentials updating of: Φ_S, Ψ_S, then Φ_{Cl_1} and Ψ_{Cl_1} using respectively equations: 3.5, 3.6, 3.8, 3.9. So, we obtain the same result as existing methods presented in chapter 2: $\delta^* = \{(D_2 = T), (D_1 = T/F)\} = 0,4$.*

3.3.2 Absorption in product-based possibilistic strong junction trees

Similar to the absorption in min-based possibilistic strong junction trees, the absorption in *product-based possibilistic strong junction trees* $\Pi\mathcal{SJT}_*^*$ allows the computation of the optimal strategy relative to the initial diagram via message passing. These messages ensure updating of the clusters and separators potentials obtained after transformation of a product-based influence diagram into $\Pi\mathcal{SJT}_*^*$.

As mentioned above, the absorption process is performed on two steps which correspond in our case to the *product-Initialization* and *product-Global-Absorption*.

3.3.2.1 Initialization in product-based possibilistic strong junction trees

The initialization of any $\Pi\mathcal{SJT}_*^*$ consists on quantifying this structure with initial possibility distributions and utility functions, that is performed similar to the qualitative case (section 3.3.1):

We assign possibility and utility potentials for each cluster, such as the possibility potential corresponds to its local joint distribution, and the utility potential depicts a numerical value associated to each strategy. These potentials are computed from the initial diagram and they can be defined respectively as follows:

Proposition 3.4. *For each cluster Cl_i we assign a possibility potential satisfying:*

$$\Phi_{Cl_i} = \Pi_p(Cl_i) \tag{3.11}$$

where $\Pi_p = \prod_{j:1..m} \pi(C_{ij}|Par(C_{ij}))$, where C_{ij} is a chance node C_j in the cluster Cl_i.

Proposition 3.5. *For each cluster Cl_i we assign an utility potential satisfying:*

$$\Psi_{Cl_i} = \sum_{i:1..n} U_i(Par'(V)) \tag{3.12}$$

where Par'(V) = Par (V) in Cl_i
and $U_i(Par'(V))$ is the utility relative to each chance or decision node in Cl_i which was a direct parent of value node V_i.

For each chance node C_j its possibility degree $\pi(C_j|Par(C_j))$ should be assigned to one and only one clique Cl_i where $dom(C_j) \subseteq dom(Cl_i)$. If this node appears in more than cluster its possibility should be assigned to the closest cluster to the leaf.

For clusters (resp. separators) without possibility potential we assign the unity potential, and for clusters (resp. separators) without utility potential we assign the null utility potential.

This process is detailed by the algorithm 3.6.

Algorithm 3.6: Product-based Initialization

Data: Initial distributions, Initial utility functions, $\Pi\mathcal{SJT}^*_*$
Result: Initialized $\Pi\mathcal{SJT}^*_*$

```
begin
    foreach cluster Cl_i do
        φ^I_{Cl_i} ← 1;
        ψ^I_{Cl_i} ← 0;
    foreach separator S_i do
        φ^I_{S_i} ← 1;
        ψ^I_{S_i} ← 0;
    foreach variable A ∈ C do
        - Choose a cluster Cl_i (a leaf or a closer cluster) containing {A} ∪ Par(A):
        φ^I_{Cl_i} ← φ^I_{Cl_i} * Π(A|Par(A));
        foreach variable A ∈ Par(V_i) do
            - Choose a cluster Cl_i (a leaf or a closer cluster) containing {A} ∪ Par(A):
            ψ^I_{Cl_i} ← ψ^I_{C_i} + U(A|Par(A));
end
```

Example 3.6. *In this example we will consider that tables 2.3, 2.4 and 2.5 present a numerical quantification of the influence diagram presented in figure 2.1. Figure 3.5 represents the initialization of the min-based strong junction tree presented in figure 3.3, where possibilities and utilities associated to clusters and separators are provided from the tables cited above.*

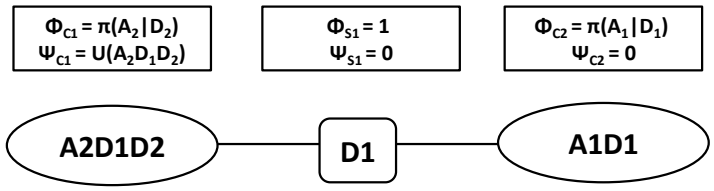

Figure 3.5: Initialization of the product-based strong junction tree

3.3.2.2 Global absorption in product-based possibilistic strong junction trees

The global absorption in $\Pi\mathcal{SJT}^*_*$ resumes the same principles of the global propagation in the min-based possibilistic strong junction trees. Namely, it's performed from leafs to the root and based on the marginalization precess defined by equation 3.4.

After the initialization, the global absorption should be performed in order to deduce the optimal strategy relative to the problem outlined by the $\Pi\mathcal{SJT}^*_*$. In our case, we have chosen to solve this problem using order of magnitude expected utility (OMEU) as decision criterion. The details relative to this procedure is presented by the algorithm 3.7.

Algorithm 3.7: Product-based Global propagation

Data: Initialized $\Pi\mathcal{SJT}^*_*$
Result: Updated $\Pi\mathcal{SJT}^*_*$
begin
- Let ***Branch*** be an ordered vector containing clusters and their separators from leaf to the root;
- **foreach** *Branch in* $\Pi\mathcal{SJT}^*_*$ **do**
  - **for** $i \leftarrow$ *1 to length(Branch)-1* **do**
    - $Cl_i \leftarrow$ Branch [i];
    - $S_k \leftarrow$ Branch [i+1];
    - $Cl_j \leftarrow$ Branch [i+2];
    - Post a message from Cl_i to S_k using equations 3.13 and 3.14;
    - Post a message from S_k to Cl_j using equations 3.15 and 3.16;

end

The equations mentioned in the algorithm 3.7 are computed as follows:

- Conditional potential of the separator:

$$\Phi_{S_i} = \sum_{Cl_i \setminus S_i}{}^{*} \Phi_{Cl_i} \tag{3.13}$$

where $\sum^*$ refers to the marginalization operator.

- Utility potential of the separator:

$$\Psi_{S_i} = \min_{Cl_i \setminus S_i} (\Psi_{Cl_i} + \Phi_{Cl_i}) \tag{3.14}$$

- Conditional potential of the cluster:

$$\Phi^{new}_{Cl_j} = \Phi_{Cl_j} * \Phi_{S_i} \tag{3.15}$$

- Utility potential of the cluster:

$$\Psi^{new}_{Cl_j} = \Psi_{Cl_j} + \frac{\Psi_{S_i}}{\Phi_{S_i}} \tag{3.16}$$

After updating potentials relative to all clusters in $\Pi\mathcal{SJT}^*_*$, the optimal strategy δ^* can be calculated using the same equation 3.10 as the case of min-based strong junction.

Example 3.7. *For the calculation of the optimal strategy of the product-based strong junction tree presented in figure 3.3 using OMEU as decision criterion, we have to compute the potentials updating of:* Φ_S, Ψ_S, *then* Φ_{Cl_1} *and* Ψ_{Cl_1} *using respectively equations: 3.13, 3.14, 3.15, 3.16. So, we obtain:* $\delta^* = \{(D_2 = F), (D_1 = T)\} = 0{,}48$.

3.4 Existing methods vs strong junction tree method to evaluate possibilistic influence diagrams

In chapter 2, we presented the existing method to evaluate possibilistic influence diagrams which are possibilistic influence diagrams (section 2.4.1) and possibilistic networks (section 2.4.2). While, in this chapter we presented a new approach to evaluate possibilistic influence diagrams using strong junction trees.

Each of these methods has advantages and disadvantages. The transformation into possibilistic decision trees maintains a clear representation of the problem. Namely, we can easily distinguish the different types of variables (i.e. chance nodes, decision nodes and value nodes) involved in the problem and also we can determinate the order of decisions. Thus, different strategies

are clearly represented which simplify the calculation of the optimal strategy. Besides, this solution was proposed for homogeneous and heterogeneous possibilistic influence diagrams. But, the major disadvantage of these decision trees that they have an exponential size for the representation of complex problems. This increase in number of nodes requires an important memory space and computation time.

For transformation into possibilistic network, the obtained structure retains the same number of nodes that the initial diagram with the dependency relationship between different variables. Besides, the solution presented in section 2.4.2 allows the evaluation of possibilistic influence diagrams with several value nodes. Thus, the calculation of the optimal strategy is made by the application of the inference algorithm. This algorithm is applied only in the case of singly connected $\mathcal{DAG}$s, while this is not always the case of obtained structures. Therefore, it is necessary to go through junction trees.

Our approach, consists on evaluating heterogeneous possibilistic influence diagrams using strong junction tree. This method with its compact representation reduces considerably the number of nodes. Then, it does not allow a clear representation of the problem. Besides, the construction of the optimal strong junction tree that corresponds to the problem requires an optimal strong triangulation which is NP-hard. Yet, the main advantage of our proposal is that it allows the evaluation of influences diagrams with cycles, which is not the case with other methods.

3.5 Implementation and experimentations

In this section we present the implementation of our method which is part of a matlab toolbox, named "Possibilistic Influence Diagrams Toolbox" (PIDT). This toolbox offers a mean of decision support under uncertainty. The main purpose of our implementation is the construction of different kinds of possibilistic influence diagrams, and then the evaluation of these models.

In this work, we propose to enrich this toolbox by the transformation method from possibilistic influence diagrams into junction trees. For this proposal, we have developed several programs regrouped by principal task.

In fact, for any type of possibilistic influence diagram (min-based or product-based) there are two principal procedures that should be implemented:

1. Graphical transformation procedure: allows the graphical transformation of possibilistic influence diagrams to possibilistic strong junction trees.
   - Moralization: Transforms the initial diagram into a moral graph.
   - Triangulation: Ensures the construction of clusters and their separators.
   - Construction of the possibilistic strong junction tree: Consists on arranging clusters and separators to form the strong junction tree with a unique root.
2. Possibilistic absorption: based on message passing from leafs to the root to determine the optimal strategy and its value with consideration to the decision criterion.
   - Initialization: allows the initialization of possibility and utility potentials for each cluster and separator.
   - Global absorption: presents computing of different equations that ensure the updating of potentials of each cluster.

This toolbox is in the process of being finalized by integration of different components and development of the graphical user interface (GUI).

3.6 Conclusion

In this chapter we have presented a new method to evaluate possibilistic influence diagrams. This method consists on a possibilistic adaptation of Jensen's method where the diagram is transformed into a strong junction tree. To do this we proceed through the graph moralization, graph triangulation, and then construction of the tree. At this level, we perform the initialization of clusters potentials (i.e. possibility and utility potentials) and then we perform the min-based absorption for determining the optimal strategy.

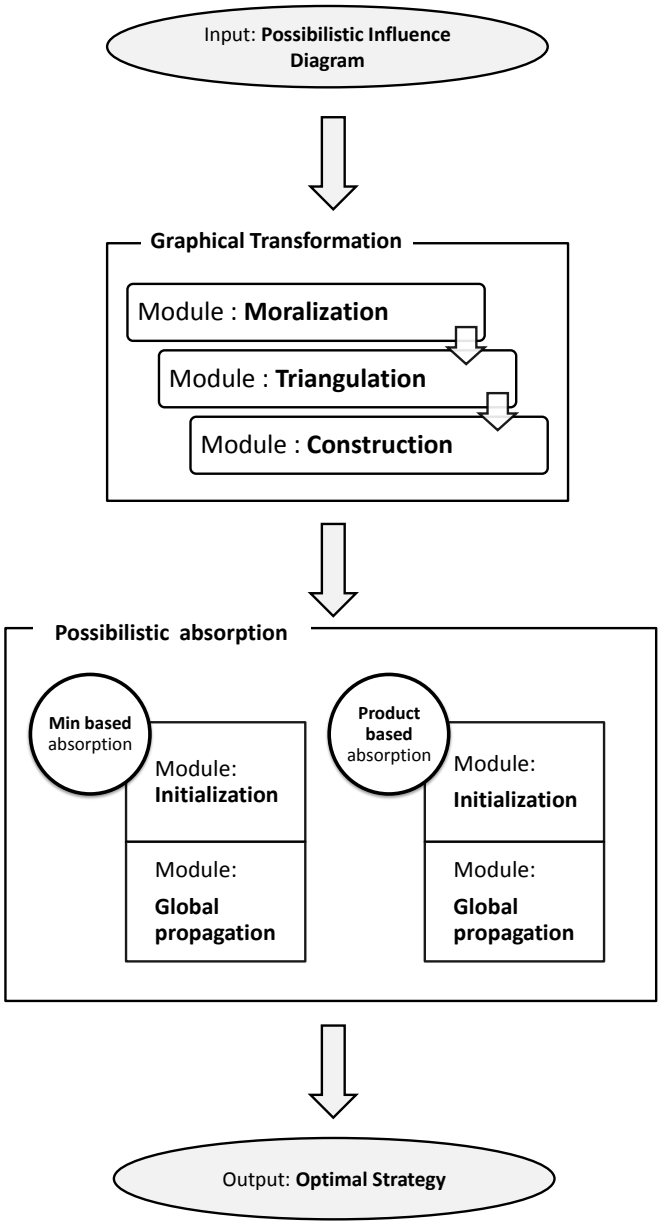

Figure 3.6: The global architecture of the possibilistic strong junction tree method

Conclusion

In this work we have proposed a new indirect method to evaluate possibilistic influence diagrams which can be applied for singly and sequential decision problems as well as multi-objective ones (i.e. problems which are characterized by more than one value node). The basics of our idea were inspired from Jensen's method[13], which is based on transformation of influence diagrams into strong junction trees (seen in chapter 1).

We chosen this approach because it differs from other ones by its ability to evaluate non-regular influence diagrams, when their structure contains cycles. So, we wanted to benefit from this advantage by adapting this method to evaluate influence diagrams, which are quantified in a possibilistic scale.

In fact, depending on the quantification of chance and value nodes, we can distinguish several kinds of possibilistic influence diagrams classified into two main categories: homogeneous influence diagrams, which encompass min-based influence diagrams and product-based influence diagrams, and heterogeneous influence diagrams. In our work we were only interested in homogeneous influence diagrams where chance and value nodes are quantified in the same scale (i.e. numerical or ordinal scale).

Moreover, decision making in possibilistic influence diagrams and evaluation of these models invoke the notion of possibilistic decision criteria presented in chapter 2. In the literature, there are panoply of possibilistic criteria, and the choice of one or other one depends essentially on the type of the influence diagram. In our case, we present evaluation of min-based influence diagrams using optimistic and pessimistic utilities [12] and evaluation of product-based influence diagrams using order of magnitude influence

diagrams [17].

As future work, we can consider direct improvements of our proposal to make it suitable for all types of possibilistic influence diagrams with several decision criteria. Besides, we aim to develop direct evaluation of these diagrams in order to provide a complete complexity study of decision making using possibilistic influence diagrams. Finally, it will be interesting to develop the possibilistic counterpart of particular classes of influence diagrams such as unconstrained influence diagrams (UIDs) and sequential influence diagrams (SIDs) [20] which can cope with asymmetric decision problems.

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