On the Relationships Between Sub Problems in the Hierarchical Optimization Framework

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Abstract. In many optimization problems there may exist multiple ways in which a particular hierarchical optimization problem can be modeled. In addition, the diversity of hierarchical optimization problems requires different types of multilevel relations between sub-problems. Thus, the approximate and accurate representations and solutions can be integrated. That is, to address the how partial solutions of sub-problems can be reintegrated to build a solution for the main problem. The nature of relations between components differs from one decomposition strategy to another. In this paper, we will investigate the possible links and relationships that may appear between sub-problems.

Keywords: Hierarchical optimization · Relationships between sub problems · Parallel processing · Sequential processing · Gradually mixed optimization · Totally mixed optimization · Stackelberg strategy

1 Introduction

Many hierarchical optimization problems can be viewed as a particular combination and ordering of other optimization problems. There, the solution of the initial problem can be rebuilt by combining solutions of its sub-problems. Generally, such sub-decisions have to be taken in a particular sequence (order) due to the fact that the solution of the upper level will define the level of optimality of its following levels. Such kind of problems was defined as hierarchical optimization problems and initially presented by Bracken and McGill [13]. Other names may be found like multi-level optimization problems [22], dynamic optimization [4]. In the hierarchical optimization problems the decision making process is divided into different dependent levels. The decisions have to be taken in a particular precedence order. That is, a decision or a solution of the first sub-problem will affect the quality of the solution found in the subsequent level. Moreover, trying to optimize the overall problem solution needs a review of all taken decision and not a solution to particular sub-problem on a particular level. The diversity of hierarchical optimization problems requires different types of multilevel relations between sub-problems. Some of them were presented in the literature. We set up the sub-problems relations framework to resume all possible relations between sub problems. We considered that relations between components of

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the master problem are divided into two basic categories: dependent sub-problems and independent sub-problems. Each category is divided into two classes. The dependent category is divided into sequential approach and parallel approach. The second category is divided into gradually mixed approach and totally mixed approaches. This paper will be organized as follows: In the next section, we will present the motivations and benefits of hierarchical optimization modeling approach. Section 3 will be devoted to detailing the relation-ships between sub-problems in the hierarchical optimization framework by presenting the Stackelberg strategy and different possible relations between components of the global problem. The proposed framework detailed in section 4 will be supported by a set of examples from the relevant literature. The paper will then be concluded and some future research perspectives will be presented in the last section.

2 Motivations and Benefits

In this section, we present a new modeling technique for complex optimization problems; it is based on the application of the Divide and Conquer strategy. The modeling process aims to identify a set of sub-problems interconnected in such a way to represent all the requirements of the main problem. The proposed optimization problems modeling alternative permits the following benefits detailed in the following subsections.

2.1 Time Minimization

The multilevel optimization consists of solving a set of sub problems and then combining the obtained partial solutions to find global solution. Sub problems are supposed to be easier to solve than the initial problem; thus the required time to solve each sub problem separately and then integrate partial solutions will be significantly less than the time required for solving the initial problem as a unit. Many works showed that in the best case using an abstraction hierarchy in problem-solving can yield an exponential speedup in search efficiency. Such a speedup is predicted by various analytical models developed in the literature and efficiency gains of this order have been confirmed empirically. This was illustrated in a number of works like the work of Bacchus and Yang [6]. Moreover, Kretinin et al. [7] modeled the problem of Fan design as a multilevel optimization problem and they showed by their experimentations that hierarchical optimization gain a considerable reduction in CPU time.

2.2 Multidisciplinary

Large-scale problems require multidisciplinary decision making at multiple levels of a decision hierarchy. The multilevel optimization facilitates the modeling of problems in which different disciplines interact. Hierarchical optimization allows designers to incorporate all relevant disciplines simultaneously. These techniques have been used in a number of fields, including automobile design, naval architecture, electronics, architecture, computers, and electricity distribution, etc. Also, such multi-discipline applications were presented in some works in literature like the work of Mesarovic et al. [16] and the work of Sobieski and Hafka [11].

2.3 Parallel Processing

Parallel processing is the ability to carry out multiple operations or tasks simultaneously. The multilevel optimization allows parallel processing in which sub problems can be solved in the same time in a parallel computing environment to guarantee a high performance computing. We can refer here to the work of Azarm and Li [19] which proves that hierarchical optimization allows parallel processing which reduces the implementation time.

2.4 Reduction of Search Space

By decomposing the initial problem into a set of sub problems we will intuitively transform the initial, generally very large search spaces into a reduced search spaces. In the literature, many works prove this motivation. As stated by Newell et al. [2] and Marvin [14], the identification of intermediate sub problems which decompose a problem can significantly reduce search and empirical evidence of the net benefit.

2.5 Reusability

After decomposing the principle problem, the resulting sub problems can be resolved iteratively or recursively by applying the same process at different levels (on different data sets). The reusability of the toolbox of programs to resolve the sub problems guarantee the consistency, extensibility and modularity.

2.6 Organization

In some cases, because of the organization of people involved in modeling and optimization, or simply for convenience, it may be easier to organize the problem as a collection of subsystems with well-defined interfaces rather than attempt to pose a single monolithic problem statement. In addition, to model complex systems, it is not possible or desirable to have a single decision-maker in charge of all decisions.

3 Relations Between Sub-problems

In this section, we will investigate the possible links and relationships that may appear between sub-problems. That is, to address the how partial solutions of sub-problems can be reintegrated to build a solution for the main problem. There are many ways in which the approximate and accurate representations and solutions can be integrated. The nature of relations between components differs from one decomposition strategy to another. In the following we present some possible relations to coordinate between sub-problems to form consistent and optimal model for the overall problem. Thus, we first introduce the Stackelberg strategy which starts from an economic point of view to demonstrate the nature of influence of sub-problem of a high level on a sub-problem a low level. Then we introduce different possible links between components of the global problem.

3.1 The Stackelberg Strategy

The Stackelberg Strategy is named by the German the economist Heinrich Freiherr von Stackelberg in 1934. In economics, the Stackelberg model is a strategic game in which the leading firm moves first and then the follower firms move sequentially. Computer science and a wide range of fields benefited from this strategy. In hierarchical optimization, the Stackelberg strategy is present in defining the type of relations between different components of the original problem. The decisions made by each sub-problem in the basic problem affect the decisions made by the others and their objectives. One set of sub-problems has the authority to strongly influence the preferences of the other sub-problems. Here we can refer to some related works in the literature; Reyniers et al. [4] examines supplier-customer interactions in quality control using the Stackelberg equilibrium approach and derives optimal strategies. According to Leitmann [8], the concept of Stackelberg strategy for a nonzero-sum two-person game is extended to allow for a non-unique rational response of the follower. They defined a generalized Stackelberg strategy, then they gave a simple example. The idea of a generalized Stackelberg strategy and strategy pair is then applied to the situation of one leader and many rational followers. Korzhyk et al. [5], attempt a study of how competition affects network efficiency by examining routing games in a flow over time model. They gave an efficiently computable Stackelberg strategy for this model (routing games in allow over time) and showed that the competitive equilibrium under this strategy is no worse than a small constant times the optimal, for two natural measures of optimality. Bhaskar et al. [23], attempt a study of how competition affects network efficiency by examining routing games in allow over time model. They present an efficiently computable Stackelberg strategy for this model and show that the competitive equilibrium under this strategy is no worse than a small constant times the optimal, for two natural measures of optimality. Also, Stackelberg strategies have been used in computer science literature to manage the efficiency loss at equilibrium like the works of Korilis et al. [24], Roughgarden [21] and Swamy [3].

3.2 Sub-problems Relationships Framework

As mentioned in the introduction of this paper, multiple relations can exist between sub-problems. The following framework present a resume of all possible relations between sub-problems which will be exhaustively detailed and argued by a set of examples in next sections.

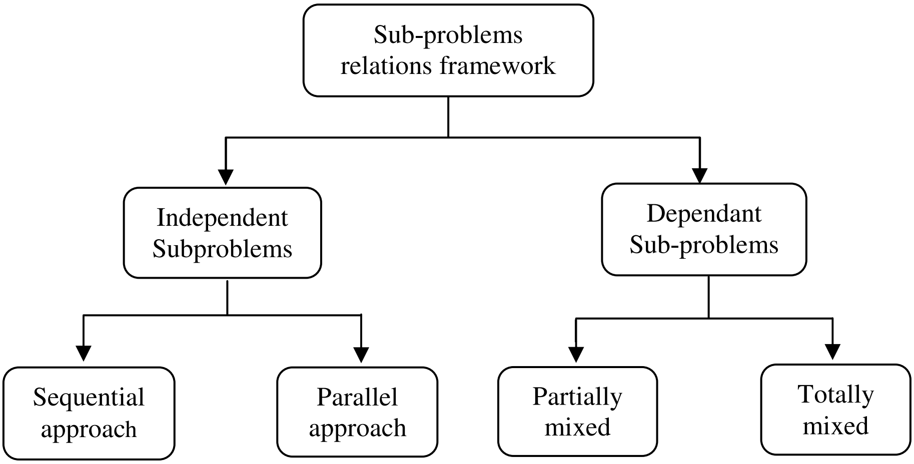

Fig. 1. Sub-problems relations framework

Independent Sub-problems Category

The independent sub-problems category supposes that each sub-problem is executed separately without any mixing with other sub-problems. Reviewing the literature, we remarked that this category was divided into two possible approaches: the sequential approach and the parallel approach. In the following two subsections we detail each approach.

Sequential Approach

In this approach, the optimization process is started using the least accurate level of representation, then after a certain set number of function evaluations, the optimization on this level is stopped and the results used as starting points for the next more accurate level. This is carried on sequentially and the number of function evaluations is decreased from one level to the next until the most accurate level is reached where fewest function evaluations are carried out. We can refer here to some works which used this approach such as the work of approach like the work of El-Beltagy and Kean [15] in which they presented empirical results. Kim et al., [9] named the same approach decomposition method and used it to cooperate between sub-problems in the Analytical Target Cascading (ATC).

Parralel Approach

In this approach, sub-problems of the principle problem interact in a collaborative form. The goal of collaboration is to allow an easy interaction amongst sub-problems from different levels. Thus, the complex problem is hierarchically decomposed into a number of sub-problems which interacts by a system-level coordination process. This form of interaction between components is well used specifically in case of multidisciplinary environment. Braun and Kroo [18] enumerate some advantageous of collaborative optimization like reducing the amount of information transferred between disciplines and removing of large iteration-loops. Reviewing the literature, we remarked that this approach has multiple terminology like “all at once approach”, “the recursive approach”, “the iterative approach” in which designers talk about a process in which top problem targets are cascaded down to the lowest level in one loop. More details are presented in the work of Kim et al. [9] in which authors presented the following figure to demonstrate the looping aspect of the parallel approach.

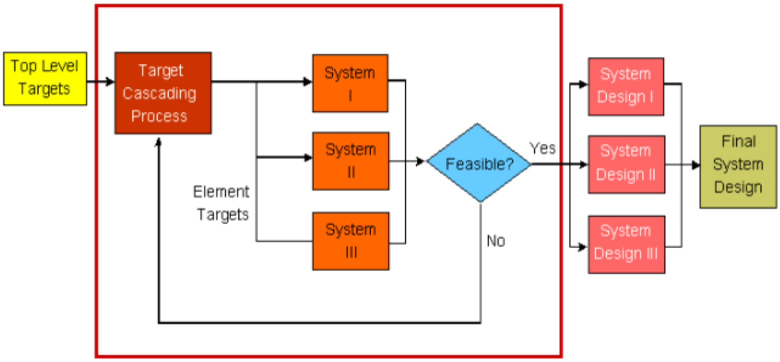

Fig. 2. Analytical Target Cascading (ATC)

Dependent Sub-problems Category

As mentioned above, this approach supposes that two or more sub-problems are executed simultaneously. Two possible ways can exist in the dependent categories which are the partially mixed multilevel approach or the totally mixed optimization approach. Each approach will be detailed in the next two subsections. Two types of mixed multilevel optimization approaches have been proposed depending on the scheduling algorithm used to organize the order and the level to reach in the solving of each sub-problem which are the gradually mixed multilevel optimization and the totally mixed optimization.

Gradually Mixed Optimization

The optimization procedure is carried out with multiple levels which are heterogeneously mixed through-out the optimization process. For example, we suppose that our initial problem is composed by three sub-problems. The master problem is gradually mixed optimized if two components are executed simultaneously in a totally mixed form and the third is executed independently. El-Baltegy and Kean [15], gives a practice example of the use of this method of relation between sub-problems, in which they presented different number of evaluations in each level.

Totally Mixed Multilevel Optimization

In totally mixed optimization, the probability of using a particular level is constant throughout most of the optimization process, i.e. all components are executed simultaneously in a totally mixed form. For example, if we suppose that our initial problem is composed by three sub-problems, if the three sub-problems are executed simultaneously in a totally mixed form then the initial optimization problem is totally mixed.

4 Illustrative Examples

In the following we try to validate the sub-problems relations framework by presenting a brief review of the current literature in models and methodologies used to model hierarchical optimization problems.

4.1 Illustrative Examples of the Parallel Approach

In this sub-section, we cite two works using the parallel approach to define the relation between different sub-problems of the master problem. Firstly, the parallel approach was well used in the ATC strategy, we can cite here works of Kim et al., [9] in the optimal design (fig. 1); Michelena et al., [17] in convergence properties; Michalek and Papalambros [12] in mechanical design; Tosserams et al., [20] in alternating directions method of multipliers. Secondly, the parallel approach was used in the work of Hertz and Lahrichi [1] illustrates parallel approach by using a data partitioning strategy in modeling a problem with a very large size (the Canadian territory), which was partitioned into 6 districts {A, B, C, D, E, F}. (fig. 3). The six districts will be executed (treated) in a parallel structure.

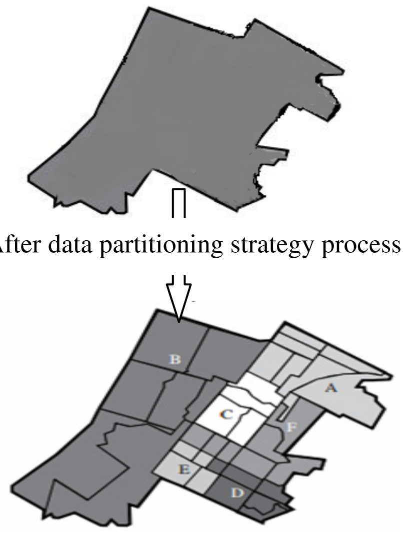

Fig. 3. The six districts of the Canadian territory

4.2 Illustrative Examples of the Sequential Approach

In this section, we basically cite the Home health care scheduling problem which a well-known combinatorial optimization problem. The home health care is an emergent kind of health care service given at home. It consists of visiting patients at their homes and performs the required treatments without a need for moving patients.

This type of service is generally given to elderly, handicapped and with special needs peoples. Patients suffering from long term maladies like Alzheimer. Moreover, some particular post-operational (after surgery) treatments can be completed at home without a need to carry them at the hospital. The hospital has a set of skilled caregivers able to perform the required task at patient homes. Basically, patients ask caregivers with particular skills to do the required treatments. Once assigned to patients, caregivers will move following specified routes to patients. The home care service dervices its importance from the considerable reduction in cost that may be incurred for patients and also for caregivers companies. The patient will have just to pay the service at home without extra charges related to hospitalization, transportation and overload of hospital facilities. The hospital or the company providing at home health care will benefit in term of patients satisfaction, low shortage rate of hospital facilities, etc. The home health care problem asks, then, for finding the set of nurses assigned to each patient and also it needs to know the routes to be followed by each team of caregivers to reach its destination. Clearly, each decision comes as answer of a particular optimization problem. The first problem is an assignment problem where the question is on which caregiver will help which patient. The second problem is on the routes to be followed by vehicles transporting nurses to reach their already assigned patients. Then, the solution of the assignment problem is an input to the routing problem. Consequently, the HCSP can be viewed and modeled as a sequential hierarchical optimization problem. The literature on the HCSP shows three types of studies of the problem: in first class the focus is on the assignment problem, in the second class of papers the main studied part is the routing problem and in some recent papers the problem is handled without omitting or hiding one of its two components. Jaber et al. [10] presented in their paper a near exhaustive literature review of the HCSP and the possible ways to model it.

4.3 Illustrative Examples of Gradually Mixed Approach

El- Baltegy and Kean [15], gives a practice example of the use of this method of relation between sub-problems, in which they presented different number of evaluations in each level. The following figure presents the rate of mixing different levels during the execution process.

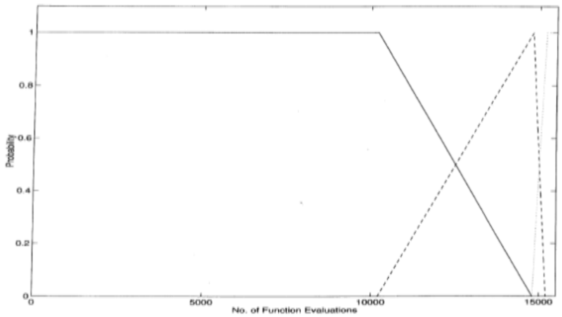

Fig. 4. Gradually mixed approach

4.4 Illustrative Examples of the Totally Mixed Approach

In the same work, El-Beltagy and Kean [15] gives an example of using the totally mixed approach in which the first level has a probability of 82.22 % of the total number of evaluations, the second has a probability of 16.44 % and the third has a probability of 1.315%.

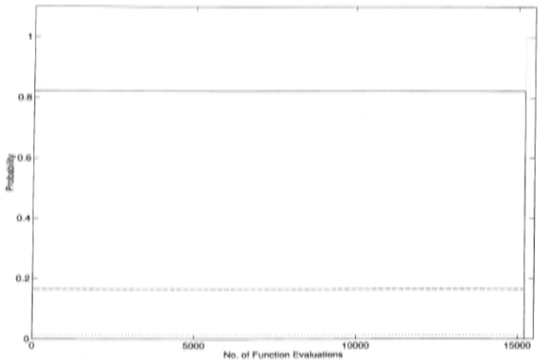

Fig. 5. Totally mixed approach

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